On the Application of Affine Resolvable Designs to Variety Trials

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Abstract

Explicit formulae for analyzing an experiment carried out in an affine resolvable block design have been given recently in Caliński and Kageyama (2008). The purpose of the present paper is to show some applications of this type of designs to experiments conducted for evaluation of crop varieties, i.e., to so-called variety trials. Attention is given to several advantages of affine resolvable designs both in planning and analyzing variety trials. Real experimental data are analyzed as examples.

The Scheme of Presentation

1. Introduction
2. A randomization-derived model for an affine resolvable design
3. Analyses based on stratum submodels
4. A combined analysis
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1. Introduction

For agricultural field experiments, especially for those with many crop varieties, the lattice designs introduced by Yates (1936, 1940) and their further extensions to the more general class of so-called generalized lattice (GL) designs have become very suitable (see Patterson and Williams, 1976; Williams, 1977; Patterson, Williams, and Hunter, 1978; Patterson and Silvey, 1980).

In the broad class of GL designs, of particular interest are the affine resolvable block designs, i.e., those in which every pair of blocks from different superblocks has the same number of treatments (varieties) in common.

Their construction and optimality properties have been considered by Bailey, Monod, and Morgan (1995). They have shown that these designs exist for various numbers of treatments (varieties) and that they are Schur-optimal among resolvable block designs, i.e., that they are optimal with respect to many criteria, including A-, D-, and E-optimality.

The statistical analysis of an experiment conducted in such an affine resolvable block design, under a randomization model, has been considered in a recent paper by Caliński and Kageyama (2008).

The purpose of the present paper is to show some applications of this type of designs to experiments conducted for evaluation of crop varieties, i.e., to so-called variety trials. Attention will be given to several advantages of these designs in planning and analyzing variety trials. This will be illustrated by examples based on real experimental data.

2. A randomization-derived model

Consider a variety trial carried out in an affine resolvable design with $v = sk$ varieties allocated in $b = sr$ blocks, each of k units (plots), grouped into r superblocks in such a way that each superblock, composed of s blocks, contains all v varieties, each of them exactly once, and that every pair of blocks from different superblocks has the same number, $k/s = k^2/v$, of varieties in common. Suppose that the randomizations of superblocks, of blocks within the superblocks and of plots within the blocks have been implemented in the trial according to the usual procedure.

In particular, the randomization of superblocks can be understood as choosing at random a permutation of numbers $1, 2, \ldots, N_A$, the original labels of available superblocks, and then renumbering them with $h = 1, 2, \ldots, N_A$, according to the order of labels in the permutation so selected. This means to use in the experiment superblocks labeled (after randomization) $h = 1, 2, \ldots, r$. In practice it will be usually $N_A = r$.

The randomization-derived model can be written as

$$\text{(2.1)} \qquad \boldsymbol{y} = \boldsymbol{\Delta}'\boldsymbol{\tau} + \boldsymbol{G}'\boldsymbol{\alpha} + \boldsymbol{D}'\boldsymbol{\beta} + \boldsymbol{\eta} + \boldsymbol{e}, \quad \text{with } \mathsf{E}(\boldsymbol{y}) = \boldsymbol{\Delta}'\boldsymbol{\tau},$$

$\boldsymbol{y} = [\boldsymbol{y}_1', \boldsymbol{y}_2', \ldots, \boldsymbol{y}_r']'$ - an $n \times 1$ vector of data concerning yield (or other variable trait) observed on $n = rv$ plots of the experiment;

$\boldsymbol{y}_h = [y_{1h}, y_{2h}, \ldots, y_{vh}]'$ - the yields observed on the v units of the super-block $h\ (= 1, 2, \ldots, r)$, ordered according to the variety labels;

$$\boldsymbol{\Delta}' = \mathbf{1}_r \otimes \boldsymbol{I}_v; \quad \boldsymbol{G}' = \boldsymbol{I}_r \otimes \mathbf{1}_v; \quad \boldsymbol{D}' = \text{diag}[\boldsymbol{D}_1' : \boldsymbol{D}_2' : \cdots : \boldsymbol{D}_r'];$$

$\boldsymbol{D}_h' = \boldsymbol{N}_h$ - $v \times s$ incidence matrix describing the hth component design (denoted by $\mathcal{D}_h$);

$\boldsymbol{\tau} = [\tau_1, \tau_2, ..., \tau_v]'$ - the variety parameters, their fixed effects;

$\boldsymbol{\alpha} = [\alpha_1, \alpha_2, ..., \alpha_r]'$ - the superblock random effects;

$\boldsymbol{\beta} = [\boldsymbol{\beta}_1', \boldsymbol{\beta}_2', ..., \boldsymbol{\beta}_r']'$, with $\boldsymbol{\beta}_h = [\beta_{1(h)}, \beta_{2(h)}, ..., \beta_{s(h)}]'$ - the block random effects;

$\boldsymbol{\eta}$, $\boldsymbol{e}$ - the $n \times 1$ vectors for the unit error and technical error random variables.

The whole block design (denoted by $\mathcal{D}^*$) can then be described by the $v \times b$ incidence matrix

$$\boldsymbol{N} = \boldsymbol{\Delta}\boldsymbol{D}' = [\boldsymbol{N}_1 : \boldsymbol{N}_2 : \cdots : \boldsymbol{N}_r],$$

where $\boldsymbol{N}'_h\boldsymbol{N}_h = k\boldsymbol{I}_s$, and $\boldsymbol{N}'_h\boldsymbol{N}_{h'} = (k/s)\mathbf{1}_s\mathbf{1}'_s$, if $h \neq h'$.

Further, note that the design (denoted by $\mathcal{D}$) by which the v varieties are assigned to the r superblocks is described by the $v \times r$ incidence matrix

$$\boldsymbol{R} = \boldsymbol{\Delta}\boldsymbol{G}' = [\mathbf{1}_v : \mathbf{1}_v : \cdots : \mathbf{1}_v],$$

i.e., it is connected and orthogonal (as is the case for any resolvable block design).

Because both $\mathcal{D}^*$ and $\mathcal{D}$ of any affine resolvable design are proper, an affine resolvable design has the orthogonal block structure property and is generally balanced (GB) in the sense of Nelder (1965).

This allows the covariance (dispersion) matrix of $\boldsymbol{y}$ to be written

(2.2) $$\text{Cov}(\boldsymbol{y}) = \boldsymbol{\phi}_1\sigma_1^2 + \boldsymbol{\phi}_2\sigma_2^2 + \boldsymbol{\phi}_3\sigma_3^2 + \boldsymbol{\phi}_4\sigma_4^2,$$

where

$$\begin{aligned} \boldsymbol{\phi}_1 &= \boldsymbol{I}_n - k^{-1}\boldsymbol{D}'\boldsymbol{D}, \quad \boldsymbol{\phi}_2 = k^{-1}\boldsymbol{D}'\boldsymbol{D} - v^{-1}\boldsymbol{G}'\boldsymbol{G}, \\ \boldsymbol{\phi}_3 &= v^{-1}\boldsymbol{G}'\boldsymbol{G} - n^{-1}\mathbf{1}_n\mathbf{1}_n', \quad \boldsymbol{\phi}_4 = n^{-1}\mathbf{1}_n\mathbf{1}_n' \end{aligned}$$

are symmetric, idempotent and pairwise orthogonal, summing to the identity matrix, and where the scalars σ_1^2, σ_2^2, σ_3^2 and σ_4^2 represent the relevant unknown stratum variances.

In the terminology of Caliński and Kageyama (2000, Section 4.4.1), any affine resolvable design belongs to the class of $(\rho_0; \rho_1; 0)$-EB designs, with the efficiency factors $\varepsilon_0 = 1$ and $\varepsilon_1 = (r-1)/r$, of multiplicities $\rho_0 = v - 1 - r(s-1)$ and $\rho_1 = r(s-1)$, respectively. Hence, the average (harmonic mean) efficiency factor of any such design is $\varepsilon = (v-1)/(\rho_0 + \varepsilon_1^{-1}\rho_1)$. This can also be seen from Theorem 3.1 in Bailey et al. (1995) and Example 3.2 in Caliński and Kageyama (2004).

3. Analyses based on stratum submodels

Considering the analysis of data from a variety trial conducted in an affine resolvable design, it will be interesting first to take into account the partial analyses based on the submodels corresponding to different strata of the experimental layout. In this, the the results given in Section 3 of Caliński and Kageyama (2008) will be helpful.

3.1. Intra-block submodel

Performing the analysis under the intra-block submodel $\boldsymbol{y}_1 = \phi_1 \boldsymbol{y}$, with $\mathrm{E}(\boldsymbol{y}_1) = \phi_1 \boldsymbol{\Delta}'\boldsymbol{\tau}$ and $\mathrm{Cov}(\boldsymbol{y}_1) = \phi_1 \sigma_1^2$, note that the C-matrix has for this design the form

(3.1) $$\boldsymbol{C}_1 = \boldsymbol{\Delta}\phi_1\boldsymbol{\Delta}' = \boldsymbol{I}_v - k^{-1}\boldsymbol{N}\boldsymbol{N}' = r(\boldsymbol{L}_0 + \varepsilon_1 \boldsymbol{L}_1),$$

of rank $v - 1$,

$$\boldsymbol{L}_0 = \boldsymbol{I}_v - v^{-1}\mathbf{1}_v\mathbf{1}_v' - \boldsymbol{L}_1 \quad \text{and} \quad \boldsymbol{L}_1 = k^{-1}\boldsymbol{N}\boldsymbol{N}' - v^{-1}r\mathbf{1}_v\mathbf{1}_v',$$

such that

$$\boldsymbol{L}_0^2 = \boldsymbol{L}_0, \quad \boldsymbol{L}_1^2 = \boldsymbol{L}_1, \quad \text{and} \quad \boldsymbol{L}_0\boldsymbol{L}_1 = \mathbf{O}.$$

Hence, a generalized inverse (g-inverse) of $\boldsymbol{C}_1$ can be obtained as $r^{-1}(\boldsymbol{L}_0 + \varepsilon_1^{-1}\boldsymbol{L}_1)$. This allows the intra-block BLUE of any contrast $\boldsymbol{c}'\boldsymbol{\tau}$ (where $\boldsymbol{c}'\mathbf{1}_v = 0$) to be

$$(\widehat{\boldsymbol{c}'\boldsymbol{\tau}})_1 = \boldsymbol{c}'\boldsymbol{C}_1^{-}\boldsymbol{Q}_1 = r^{-1}\boldsymbol{c}'[\boldsymbol{I}_v + (r-1)^{-1}k^{-1}\boldsymbol{N}\boldsymbol{N}']\boldsymbol{Q}_1,$$

$$\boldsymbol{Q}_1 = \boldsymbol{\Delta}\boldsymbol{\phi}_1\boldsymbol{y} = \sum_{h=1}^{r}(\boldsymbol{I}_v - k^{-1}\boldsymbol{D}_h'\boldsymbol{D}_h)\boldsymbol{y}_h = \sum_{h=1}^{r}(\boldsymbol{I}_v - k^{-1}\boldsymbol{N}_h\boldsymbol{N}_h')\boldsymbol{y}_h,$$

and

$$\begin{aligned}\text{Var}[(\widehat{\boldsymbol{c}'\boldsymbol{\tau}})_1] = \boldsymbol{c}'\boldsymbol{C}_1^{-}\boldsymbol{c}\sigma_1^2 &= r^{-1}\boldsymbol{c}'(\boldsymbol{L}_0 + \varepsilon_1^{-1}\boldsymbol{L}_1)\boldsymbol{c}\sigma_1^2 \\ &= r^{-1}\boldsymbol{c}'[\boldsymbol{I}_v + (r-1)^{-1}k^{-1}\boldsymbol{N}\boldsymbol{N}']\boldsymbol{c}\sigma_1^2, \qquad (3.2)\end{aligned}$$

σ_1^2 being the intra-block stratum variance.

In particular, for any elementary contrast,

$$\tau_i - \tau_{i'}, \quad i, i' = 1, 2, ..., v \ (i \neq i'),$$

the variance of its intra-block BLUE gets the form

$$\text{Var}[(\widehat{\tau_i - \tau_{i'}})_1] = 2[\frac{1}{r} + \frac{r - \lambda_{ii'}}{rk(r-1)}]\sigma_1^2, \tag{3.3}$$

where $\lambda_{ii'}$ is the number of blocks in which the ith and the i'th varieties “concur”, i.e., are both present (see also Theorem 3.6 in Bailey et al., 1995).

Usually of interest are also estimators of the variety main effects, $\tau_i - \tau.$ (where $\tau. = v^{-1}\sum_{i=1}^{v}\tau_i$), for which

$$\text{Var}[(\widehat{\tau_i - \tau.})_1] = \frac{1}{v}[\frac{v-1}{r} + \frac{v-k}{k(r-1)}]\sigma_1^2 \quad (= \sigma^2_{(\widehat{\tau_i - \tau.})_1}, \ \text{say}),$$

constant for all varieties, i.e., for any $i \ (= 1, 2, ..., v)$.

As to the analysis of variance (ANOVA) under this submodel, it can be written

$$\boldsymbol{y}'\boldsymbol{\phi}_1\boldsymbol{y} = \boldsymbol{Q}_1'\boldsymbol{C}_1^{-}\boldsymbol{Q}_1 + \boldsymbol{y}'\boldsymbol{\psi}_1\boldsymbol{y}, \quad \text{with} \quad \boldsymbol{\psi}_1 = \boldsymbol{\phi}_1 - \boldsymbol{\phi}_1\boldsymbol{\Delta}'\boldsymbol{C}_1^{-}\boldsymbol{\Delta}\boldsymbol{\phi}_1,$$

where, for the considered design,

$$\boldsymbol{Q}_1'\boldsymbol{C}_1^{-}\boldsymbol{Q}_1 = r^{-1}\boldsymbol{Q}_1'(\boldsymbol{L}_0+\varepsilon_1^{-1}\boldsymbol{L}_1)\boldsymbol{Q}_1 = r^{-1}\boldsymbol{Q}_1'[\boldsymbol{I}_v+(r-1)^{-1}k^{-1}\boldsymbol{N}\boldsymbol{N}']\boldsymbol{Q}_1$$

is the intra-block treatment (variety) sum of squares, on $v-1$ degrees of freedom (d.f.),

$$\boldsymbol{y}'\boldsymbol{\psi}_1\boldsymbol{y} = \boldsymbol{y}'\boldsymbol{\phi}_1\boldsymbol{y} - r^{-1}\boldsymbol{Q}_1'[\boldsymbol{I}_v + (r-1)^{-1}k^{-1}\boldsymbol{N}\boldsymbol{N}']\boldsymbol{Q}_1$$

is the intra-block residual sum of squares, on $d_1 = n - b - v + 1$ d.f., giving

$$s_1^2 = d_1^{-1}\boldsymbol{y}'\boldsymbol{\psi}_1\boldsymbol{y}, \tag{3.4}$$

which (under this submodel) is the MINQUE of σ_1^2. It can be used to obtain an unbiased estimator of Var$[(\widehat{\boldsymbol{c}'\boldsymbol{\tau}})_1]$, replacing σ_1^2 there by s_1^2.

This ANOVA, assuming that $\boldsymbol{y}$ has a multivariate normal distribution, offers a variance ratio criterion for testing

$$H_{01} : \boldsymbol{\tau}'\boldsymbol{C}_1\boldsymbol{\tau} = 0,$$

equivalent to E$(\boldsymbol{y}_1) = \boldsymbol{0}$ [i.e., $\boldsymbol{\Delta}'\boldsymbol{\tau} = k^{-1}\boldsymbol{D}'\boldsymbol{D}\boldsymbol{\Delta}'\boldsymbol{\tau}$, which is equivalent to $\boldsymbol{\tau} = (kr)^{-1}\boldsymbol{N}\boldsymbol{N}'\boldsymbol{\tau}$].

Such criterion,

$$F_1 = (v-1)^{-1}\boldsymbol{Q}_1'\boldsymbol{C}_1^{-}\boldsymbol{Q}_1/s_1^2,$$

has a noncentral F distribution with $v-1$ and d_1 d.f., and the noncentrality parameter $\delta_1 = \boldsymbol{\tau}'\boldsymbol{C}_1\boldsymbol{\tau}/\sigma_1^2$, central if the hypothesis is true (see also Ceranka, 1975).

If H_{01} is rejected, one may test hypotheses

$$H_{01,c'\tau} : \boldsymbol{c}'\boldsymbol{\tau} = 0 \quad (\text{where } \boldsymbol{c}'\mathbf{1}_v = 0),$$

using the statistic

$$F_{1,c'\tau} = [(\widehat{\boldsymbol{c}'\boldsymbol{\tau}})_1]^2/\widehat{\text{Var}}[(\widehat{\boldsymbol{c}'\boldsymbol{\tau}})_1],$$

where

$$\widehat{\text{Var}}[(\widehat{\boldsymbol{c}'\boldsymbol{\tau}})_1] = r^{-1}\boldsymbol{c}'[\boldsymbol{I}_v + (r-1)^{-1}k^{-1}\boldsymbol{N}\boldsymbol{N}']\boldsymbol{c}s_1^2.$$

It has a noncentral F distribution with 1 and d_1 d.f., and the noncentrality parameter

$$\delta_{1,c'\tau} = (\boldsymbol{c}'\boldsymbol{\tau})^2/\text{Var}[(\widehat{\boldsymbol{c}'\boldsymbol{\tau}})_1],$$

the distribution being central when $H_{01,c'\tau}$ is true.

3.2. Inter-block-intra-superblock submodel

Going to the analysis under the inter-block-intra-superblock submodel $\boldsymbol{y}_2 = \phi_2 \boldsymbol{y}$, with E$(\boldsymbol{y}_2) = \phi_2 \boldsymbol{\Delta}' \boldsymbol{\tau}$ and Cov$(\boldsymbol{y}_2) = \phi_2 \sigma_2^2$, note that the relevant C-matrix is

(3.5) $$\boldsymbol{C}_2 = k^{-1}\boldsymbol{N}\boldsymbol{N}' - v^{-1} r \mathbf{1}_v \mathbf{1}_v' = \boldsymbol{L}_1, \quad \text{of rank } b - r.$$

Thus, as its g-inverse one can take $\boldsymbol{I}_v$. From this, the inter-block-intra--superblock BLUE of any contrast $\boldsymbol{c}'\boldsymbol{\tau}$ such that $\boldsymbol{c} = \boldsymbol{C}_2 \boldsymbol{s}$, for some $\boldsymbol{s}$, is

$$(\widehat{\boldsymbol{c}'\boldsymbol{\tau}})_2 = \boldsymbol{c}'\boldsymbol{C}_2^- \boldsymbol{Q}_2 = \boldsymbol{c}'\boldsymbol{Q}_2,$$

$$\boldsymbol{Q}_2 = \boldsymbol{\Delta}\phi_2 \boldsymbol{y} = \sum_{h=1}^{r} (k^{-1}\boldsymbol{D}_h'\boldsymbol{D}_h - v^{-1}\mathbf{1}_v\mathbf{1}_v')\boldsymbol{y}_h = \sum_{h=1}^{r} (k^{-1}\boldsymbol{N}_h\boldsymbol{N}_h' - v^{-1}\mathbf{1}_v\mathbf{1}_v')\boldsymbol{y}_h,$$

and its variance

$$\text{Var}[(\widehat{\boldsymbol{c}'\boldsymbol{\tau}})_2] = \boldsymbol{c}'\boldsymbol{C}_2^- \boldsymbol{c}\sigma_2^2 = \boldsymbol{c}'\boldsymbol{c}\sigma_2^2,$$

where σ_2^2 is the inter-block-intra-superblock stratum variance.

Note here that the general formula for the inter-block--intra-superblock ANOVA,

$$\boldsymbol{y}'\boldsymbol{\phi}_2\boldsymbol{y} = \boldsymbol{Q}_2'\boldsymbol{C}_2^{-}\boldsymbol{Q}_2 + \boldsymbol{y}'\boldsymbol{\psi}_2\boldsymbol{y}$$

with

$$\boldsymbol{\psi}_2 = \boldsymbol{\phi}_2 - \boldsymbol{\phi}_2\boldsymbol{\Delta}'\boldsymbol{C}_2^{-}\boldsymbol{\Delta}\boldsymbol{\phi}_2,$$

reduces, for the considered designs, to

$$\boldsymbol{y}'\boldsymbol{\phi}_2\boldsymbol{y} = \boldsymbol{Q}_2'\boldsymbol{C}_2^{-}\boldsymbol{Q}_2 = \boldsymbol{Q}_2'\boldsymbol{Q}_2, \quad \text{because } \boldsymbol{\psi}_2 = \mathbf{O}.$$

This means that this analysis is reduced to the inter--block-intra-superblock treatment (variety) sum of squares, $\boldsymbol{Q}_2'\boldsymbol{Q}_2$, on rank$(\boldsymbol{L}_1) = b - r$ d.f. Thus, it provides no residuals for estimating the stratum variance σ_2^2.

3.3. Inter-superblock submodel

Under the inter-superblock submodel

$$\boldsymbol{y}_3 = \boldsymbol{\phi}_3 \boldsymbol{y}, \quad \text{with} \quad \mathsf{E}(\boldsymbol{y}_3) = \boldsymbol{\phi}_3 \boldsymbol{\Delta}' \boldsymbol{\tau} \quad \text{and} \quad \mathsf{Cov}(\boldsymbol{y}_3) = \boldsymbol{\phi}_3 \sigma_3^2,$$

for which the relevant C-matrix is reduced to

$$\boldsymbol{C}_3 = v^{-1} \boldsymbol{R}\boldsymbol{R}' - v^{-1} r \mathbf{1}_v \mathbf{1}_v' = \mathbf{O}.$$

This implies that for no function $\boldsymbol{c}'\boldsymbol{\tau}$ (with $\boldsymbol{c} \neq \mathbf{0}$) the BLUE under this submodel exists. Hence, the analysis under this submodel provides no information on contrasts of variety parameters. It is interesting only for estimating the stratum variance σ_3^2, taking

(3.6) $$s_3^2 = (r-1)^{-1} \boldsymbol{y}' \boldsymbol{\psi}_3 \boldsymbol{y} = (r-1)^{-1} \boldsymbol{y}' \boldsymbol{\phi}_3 \boldsymbol{y},$$

$$\boldsymbol{\psi}_3 = \boldsymbol{\phi}_3 = (\boldsymbol{I}_r - r^{-1} \mathbf{1}_r \mathbf{1}_r') \otimes v^{-1} \mathbf{1}_v \mathbf{1}_v' \quad \text{is of rank} \;\; r-1.$$

3.4. Total-area submodel

Coming finally to the to the total area submodel

$$\boldsymbol{y}_4 = \boldsymbol{\phi}_4 \boldsymbol{y}, \quad \text{with} \quad \mathsf{E}(\boldsymbol{y}_4) = \boldsymbol{\phi}_4 \boldsymbol{\Delta}' \boldsymbol{\tau} \quad \text{and} \quad \mathsf{Cov}(\boldsymbol{y}_4) = \boldsymbol{\phi}_4 \sigma_4^2,$$

note that it is interesting only for obtaining the BLUE of the general parametric mean, $\boldsymbol{c}'\boldsymbol{\tau} = v^{-1}\mathbf{1}_v'\boldsymbol{\tau}$, which is simply

$$(\widehat{\boldsymbol{c}'\boldsymbol{\tau}})_4 = n^{-1}\mathbf{1}_n'\boldsymbol{y}, \quad \text{with} \;\; \mathsf{Var}[(\widehat{\boldsymbol{c}'\boldsymbol{\tau}})_4] = n^{-1}\sigma_4^2.$$

However, because no residuals are left under this submodel, no estimation of σ_4^2 is under it available (see also Caliński and Kageyama, 2000, Section 5.3.4).

4. A combined analysis

With results from the four strata, one becomes interested in combining them in a way corresponding to the overall model (2.1), with

$$\text{Cov}(\boldsymbol{y}) = \boldsymbol{\phi}_1\sigma_1^2 + \boldsymbol{\phi}_2\sigma_2^2 + \boldsymbol{\phi}_3\sigma_3^2 + \boldsymbol{\phi}_4\sigma_4^2.$$

First suppose, provisionally, that the stratum variances σ_1^2 and σ_2^2 are known. Then, for $\boldsymbol{\tau}$, one obtains the BLUE

$$\hat{\boldsymbol{\tau}} = r^{-1}(\boldsymbol{L}_0\boldsymbol{Q}_1 + \varepsilon_{11}^{-1}w_1\boldsymbol{L}_1\boldsymbol{Q}_1 + \varepsilon_{21}^{-1}w_2\boldsymbol{L}_1\boldsymbol{Q}_2) + n^{-1}\mathbf{1}_v\mathbf{1}_n'\boldsymbol{y}, \tag{4.1}$$

$$\varepsilon_{11} = \varepsilon_1 = (r-1)/r,\ \varepsilon_{21} = 1 - \varepsilon_1 = 1/r,$$

$$w_1 = \frac{\varepsilon_{11}\sigma_2^2}{\varepsilon_{11}\sigma_2^2 + \varepsilon_{21}\sigma_1^2} \quad \text{and} \quad w_2 = \frac{\varepsilon_{21}\sigma_1^2}{\varepsilon_{11}\sigma_2^2 + \varepsilon_{21}\sigma_1^2}. \tag{4.2}$$

The covariance matrix of $\hat{\boldsymbol{\tau}}$ is

$$\text{Cov}(\widehat{\boldsymbol{\tau}}) = r^{-1}(\sigma_1^2\boldsymbol{L}_0 + \sigma_1^2\varepsilon_{11}^{-1}w_1\boldsymbol{L}_1 + \sigma_4^2 v^{-1}\mathbf{1}_v\mathbf{1}_v'), \tag{4.3}$$

giving $\text{Var}(\widehat{\boldsymbol{c}'\boldsymbol{\tau}}) = r^{-1}\boldsymbol{c}'(\boldsymbol{L}_0 + \varepsilon_{11}^{-1}w_1\boldsymbol{L}_1)\boldsymbol{c}\sigma_1^2$ for the BLUE of any contrast $\boldsymbol{c}'\boldsymbol{\tau}$ ($\boldsymbol{c}'\mathbf{1}_v = 0$).

With regard to $\hat{\boldsymbol{\tau}}$, two particular cases are worth mentioning.

First, note that if $\sigma_1^2 \to 0$, then

$$\hat{\boldsymbol{\tau}} \to r^{-1}(\boldsymbol{L}_0 + \varepsilon_{11}^{-1}\boldsymbol{L}_1)\boldsymbol{Q}_1 + n^{-1}\mathbf{1}_v\mathbf{1}_n'\boldsymbol{y},$$

i.e., the contribution of the inter-block-intra-superblock stratum is negligible.

Another case of interest is $\sigma_1^2 = \sigma_2^2$. It can be shown that then

$$\hat{\boldsymbol{\tau}} = r^{-1}\sum_{h=1}^{r}(\boldsymbol{I}_v - v^{-1}\mathbf{1}_v\mathbf{1}_v')\boldsymbol{y}_h + n^{-1}\mathbf{1}_v\mathbf{1}_n'\boldsymbol{y} = r^{-1}\sum_{h=1}^{r}\boldsymbol{y}_h,$$

which means that the BLUE of $\boldsymbol{\tau}$ reduces to ordinary variety means, as in a randomized complete block design.

In practice, the variances σ_1^2 and σ_2^2 appearing in w_1 and w_2 are usually unknown and need to be estimated. As shown in Caliński and Kageyama (2008), they can be estimated, following Nelder (1968), using

(4.4) $$\hat{\sigma}^2_{1(N)} = d_1^{-1}\boldsymbol{y}'\boldsymbol{\psi}_1\boldsymbol{y} \quad (= s_1^2)$$

and

(4.5) $$\hat{\sigma}^2_{2(N)} = \frac{1}{\rho_1}[\frac{\varepsilon_{21}}{r}(\varepsilon_{11}^{-1}\boldsymbol{Q}_1' - \varepsilon_{21}^{-1}\boldsymbol{Q}_2')\boldsymbol{L}_1(\varepsilon_{11}^{-1}\boldsymbol{Q}_1 - \varepsilon_{21}^{-1}\boldsymbol{Q}_2) - \frac{\varepsilon_{21}}{\varepsilon_{11}}\,\frac{\rho_1}{d_1}\boldsymbol{y}'\boldsymbol{\psi}_1\boldsymbol{y}] \quad (= s_2^2, \text{ say}).$$

With them,

(4.6) $$\hat{w}_1 = \frac{\varepsilon_{11}\hat{\sigma}^2_{2(N)}}{\varepsilon_{11}\hat{\sigma}^2_{2(N)} + \varepsilon_{21}\hat{\sigma}^2_{1(N)}} \quad \text{and} \quad \hat{w}_2 = \frac{\varepsilon_{21}\hat{\sigma}^2_{1(N)}}{\varepsilon_{11}\hat{\sigma}^2_{2(N)} + \varepsilon_{21}\hat{\sigma}^2_{1(N)}}.$$

Note that these estimates are obtainable directly, not by an iterative procedure. This is an important advantage of affine resolvable designs.

Note that these Nelder estimators coincide here with those obtainable in the classic Yates (1939) – Rao (1956) approach, and are exactly the same as those obtainable by the MML (REML) method of Patterson and Thompson (1971, 1975), which in turn coincide with those obtainable by the (iterated) MINQUE procedure of Rao (1972, 1979). This implies that the intra-block residual mean square $\hat{\sigma}^2_{1(N)} = s^2_1$ is the MINQUE of σ^2_1 not only under the intra-block submodel but also under the overall model (2.1).

It may be added, that the inter-block-intra-superblock stratum variance $\sigma^2_2,$ for which there is no estimator in the analysis under the submodel corresponding to that stratum, receives now the MINQUE, $\hat{\sigma}^2_{2(N)} = s^2_2,$ under the overall model (2.1). Also the inter-superblock mean square s^2_3 can be seen as obtainable from the relevant Nelder (1968) equation, reduced for any affine resolvable design to

$$\boldsymbol{y}'\boldsymbol{\psi}_3\boldsymbol{y} = \sigma^2_3 d'_3, \quad \text{with} \;\; d'_3 = r - 1,$$

being at the same time the MINQUE of σ^2_3 under (2.1).

Now note that when the weights w_1 and w_2 in the formula for $\hat{\boldsymbol{\tau}}$ are replaced by $\hat{w}_1$ and $\hat{w}_2$, one obtains an empirical estimator

$$\tilde{\boldsymbol{\tau}} = r^{-1}(\boldsymbol{L}_0\boldsymbol{Q}_1 + \varepsilon_{11}^{-1}\hat{w}_1\boldsymbol{L}_1\boldsymbol{Q}_1 + \varepsilon_{21}^{-1}\hat{w}_2\boldsymbol{L}_1\boldsymbol{Q}_2) + n^{-1}\mathbf{1}_v\mathbf{1}_n'\boldsymbol{y},$$

with the properties

$$\mathsf{E}(\tilde{\boldsymbol{\tau}}) = \mathsf{E}(\hat{\boldsymbol{\tau}}) = \boldsymbol{\tau},$$

(4.7) $$\mathsf{Cov}(\tilde{\boldsymbol{\tau}}) \cong r^{-1}[\sigma_1^2\boldsymbol{L}_0 + \sigma_1^2\varepsilon_{11}^{-1}w_1\boldsymbol{L}_1(1+\zeta) + \sigma_4^2 v^{-1}\mathbf{1}_v\mathbf{1}_v'],$$

(4.8) $$\zeta = \frac{2(n-v-r+1)}{(b-r)(n-v-b+1)}\,\frac{w_2}{w_1},$$

as it follows from the approximation suggested by Kackar and Harville (1984). This implies that for any contrast $\boldsymbol{c}'\boldsymbol{\tau}$ the empirical estimator is of the form

(4.9) $$\widetilde{\boldsymbol{c}'\boldsymbol{\tau}} = r^{-1}\boldsymbol{c}'(\boldsymbol{L}_0\boldsymbol{Q}_1 + \varepsilon_{11}^{-1}\hat{w}_1\boldsymbol{L}_1\boldsymbol{Q}_1 + \varepsilon_{21}^{-1}\hat{w}_2\boldsymbol{L}_1\boldsymbol{Q}_2)$$

with

(4.10) $$\begin{aligned}\mathsf{Var}(\widetilde{\boldsymbol{c}'\boldsymbol{\tau}}) &\cong r^{-1}\boldsymbol{c}'[\boldsymbol{L}_0 + \varepsilon_{11}^{-1}w_1\boldsymbol{L}_1(1+\zeta)]\boldsymbol{c}\sigma_1^2 = \\ &= \mathsf{Var}(\widehat{\boldsymbol{c}'\boldsymbol{\tau}}) + r^{-1}\varepsilon_{11}^{-1}w_1\zeta\boldsymbol{c}'\boldsymbol{L}_1\boldsymbol{c}\sigma_1^2.\end{aligned}$$

If of interest are estimators of elementary contrasts, their variance is

$$\text{Var}(\widetilde{\tau_i - \tau_{i'}}) \cong 2\left\{\frac{1}{r} + \frac{r[w_1(1+\zeta)-1]+1}{rk(r-1)}(r-\lambda_{ii'})\right\}\sigma_1^2,$$

which reduces to $\text{Var}[(\widehat{\boldsymbol{c}'\boldsymbol{\tau}})_1]$ in the extreme case of $w_1 = 1$ and, hence, $\zeta = 0$.

For estimators of variety main effects, the relevant variance is

$$\text{Var}(\widetilde{\tau_i - \tau_.}) \cong \frac{1}{v}\left[\frac{v-1}{r} + \frac{w_1(1+\zeta)-\varepsilon_{11}}{\varepsilon_{11}}\,\frac{v-k}{k}\right]\sigma_1^2,$$

$$(= \sigma^2_{\widetilde{\tau_i - \tau_.}}, \;\; \text{say})$$

constant for any i $(= 1, 2, ..., v)$.

Evidently, Var$(\widetilde{\boldsymbol{c}'\boldsymbol{\tau}})$ reduces to Var$(\widehat{\boldsymbol{c}'\boldsymbol{\tau}})$ if $\boldsymbol{L}_1\boldsymbol{c} = \boldsymbol{0}$. On the other hand, if $\boldsymbol{L}_0\boldsymbol{c} = \boldsymbol{0}$,

(4.11) $$\text{Var}(\widetilde{\boldsymbol{c}'\boldsymbol{\tau}}) \cong r^{-1}\sigma_1^2 w_1 \varepsilon_{11}^{-1}\boldsymbol{c}'\boldsymbol{L}_1\boldsymbol{c}(1+\zeta) = \text{Var}[(\widehat{\boldsymbol{c}'\boldsymbol{\tau}})_1]w_1(1+\zeta).$$

This shows that the approximate variance (4.11) is smaller than Var$[(\widehat{\boldsymbol{c}'\boldsymbol{\tau}})_1]$ if and only if $\zeta < w_2/w_1$, which in practice holds in most cases. If in a particular case it does not hold, or the ratio w_2/w_1, and hence ζ, is close to 0, it may be reasonable to replace $\hat{w}_1$ by 1 and, hence, $\hat{w}_2$ by 0, i.e., to use $(\widehat{\boldsymbol{c}'\boldsymbol{\tau}})_1$, from the intra-block analysis, instead of $\widetilde{\boldsymbol{c}'\boldsymbol{\tau}}$.

Suppose now that one is interested in testing

$$H_0 : \tau_1 = \tau_2 = \cdots = \tau_v.$$

It can be tested approximately using

$$F = \frac{d_1}{v-1} \, \frac{\text{SS}_0 + \text{SS}_1}{\boldsymbol{y}'\boldsymbol{\psi}_1\boldsymbol{y}}, \tag{4.12}$$

where

$$\text{SS}_0 = r^{-1}\boldsymbol{Q}_1'\boldsymbol{L}_0\boldsymbol{Q}_1,$$

$$\text{SS}_1 = r^{-1}\varepsilon_{11}[\hat{w}_1(1+\hat{\zeta})]^{-1}(\hat{w}_1\varepsilon_{11}^{-1}\boldsymbol{Q}_1' + \hat{w}_2\varepsilon_{21}^{-1}\boldsymbol{Q}_2')\boldsymbol{L}_1(\hat{w}_1\varepsilon_{11}^{-1}\boldsymbol{Q}_1 + \hat{w}_2\varepsilon_{21}^{-1}\boldsymbol{Q}_2).$$

Assuming the multivariate normal distribution of $\boldsymbol{y}$, the unknown distribution of (4.12) can be, under H_0, justifiably approximated by the central F distribution with d and $d_1 = n - b - v + 1$ d.f., where

$$d = \frac{2(v-1)^2}{2(v-1) + \dfrac{\zeta(w_2 - 3w_1\zeta)^2\rho_1(\rho_1+2)}{w_1 w_2 (1+\zeta)^2}}. \tag{4.13}$$

In practice, the unknown weights, w_1 and w_2, in (4.13) have to be replaced by their estimates.

If H_0 is rejected, one may be interested in finding which of the implied hypotheses are responsible for that. If $\boldsymbol{c}'\boldsymbol{\tau}$ $(\boldsymbol{c}'\mathbf{1}_v = 0)$ is of interest, then the relevant hypothesis is

$$H_{0,c'\tau} : \boldsymbol{c}'\boldsymbol{\tau} = 0.$$

It can be tested approximately using

$$F_{c'\tau} = (\widetilde{\boldsymbol{c}'\boldsymbol{\tau}})^2 / \widehat{\text{Var}}(\widetilde{\boldsymbol{c}'\boldsymbol{\tau}}), \tag{4.14}$$

$$\begin{aligned}
\widehat{\text{Var}}(\widetilde{\boldsymbol{c}'\boldsymbol{\tau}}) &= r^{-1}\boldsymbol{c}'[\boldsymbol{L}_0 + \varepsilon_{11}^{-1}\hat{w}_1\boldsymbol{L}_1(1+\hat{\zeta})]\boldsymbol{c}s_1^2 \\
&= r^{-1}\{\boldsymbol{c}'\boldsymbol{c} + k^{-1}[\varepsilon_{11}^{-1}\hat{w}_1(1+\hat{\zeta}) - 1]\boldsymbol{c}'\boldsymbol{N}\boldsymbol{N}'\boldsymbol{c}\}s_1^2,
\end{aligned}$$

The unknown distribution of (4.14) can, under $H_{0,c'\tau}$, be approximated by the central F distribution with 1 and d_1 d.f. For the theory underlying the proposed tests see Sections 3.8.5 and 5.5.4 in Caliński and Kageyama (2000).

5. Applications

Affine resolvable designs are often used in variety trials. In particular, they are employed quite often in designing variety trials organized by the Research Centre for Cultivar Testing in Poland. The data analyzed by the presented methodology come from three series of variety trials carried out, under the auspices of that Research Centre, at various Experimental Stations for Variety Testing spread over the country.

5.1. Winter wheat variety trials

The first series is composed of 13 trials conducted in the 1997/1998 season with varieties of winter wheat. Each trial was arranged in an affine resolvable design with $v = 32$ varieties allocated in $b = 16$ blocks grouped into $r = 4$ superblocks (replications), each containing $s = 4$ blocks of size $k = 8$. The varieties were assigned to blocks in such a way that every pair of blocks from different superblocks had the same number, $k/s = k^2/v = 2$, of varieties in common, satisfying the condition for a resolvable design to be affine resolvable. The average (harmonic mean) efficiency factor of the design was $\varepsilon = 0.8857$.

The layout of the design (before randomization) is shown in Table 1.

Table 1: The design used in the winter wheat variety trials, in its prerandomization form

Block	Superblock 1 Varieties								Block	Superblock 2 Varieties							
1	1	5	9	13	17	21	25	29	1	1	6	11	13	18	23	28	32
2	2	6	10	14	18	22	26	30	2	2	7	12	14	19	24	25	29
3	3	7	11	15	19	23	27	31	3	3	8	9	15	20	21	26	30
4	4	8	12	16	20	24	28	32	4	4	5	10	16	17	22	27	31

Block	Superblock 3 Varieties								Block	Superblock 4 Varieties							
1	1	7	10	15	17	24	26	32	1	1	6	12	15	20	22	27	29
2	2	8	11	16	18	21	27	29	2	2	7	9	16	17	23	28	30
3	3	5	12	13	19	22	28	30	3	3	8	10	13	18	24	25	31
4	4	6	9	14	20	23	25	31	4	4	5	11	14	19	21	26	32

To each of the trials the analyses described in Sections 3 and 4 have been applied, to data concerning the variety plot grain yields (expressed in dt/ha).

First of all, it is interesting to look at the estimates of the stratum variances σ_1^2, σ_2^2, and σ_3^2. According to one of the basic principles of experimental design, the “local control" (see, e.g., Yates, 1965), the experiment should be arrange in the field in such a way that the plots within the blocks are as uniform as possible, allowing for larger variation between blocks within superblocks, and even larger between the superblocks within the experimental field.

If this is done successfully, in the sense of cotrolling the soil variation properly, the estimated stratum variances should satisfy the relation $s_1^2 < s_2^2 < s_3^2$.

In fact, because the inter-superblock stratum provides no information on contrasts of variety parameters, the relation $s_1^2 < s_2^2$ is essential. It is satisfied by all trials except one. See Table 2.

Table 2: Main results characterizing the individual trials conducted in 1997/1998 with varieties of winter wheat

Loc.	s_1^2	s_2^2	s_3^2	s_1^2/s_2^2	$\hat{w}_1$	$\hat{w}_2$	$\hat{w}_2/\hat{w}_1$	$\hat{\zeta}$	$1-\hat{\sigma}^2_{\widetilde{\tau_i-\tau.}}/\hat{\sigma}^2_{\widehat{(\tau_i-\tau.)}_1}$
1	14.0916	56.1499	164.3882	0.2510	0.9228	0.0772	0.0837	0.0160	0.0285
2	7.3535	48.0084	47.2961	0.1532	0.9514	0.0486	0.0511	0.0098	0.0180
3	21.5460	57.9498	296.9062	0.3718	0.8897	0.1103	0.1239	0.0237	0.0408
4	6.3335	33.7029	43.9610	0.1879	0.9411	0.0589	0.0626	0.0120	0.0218
5	12.0534	32.4167	87.3901	0.3718	0.8897	0.1103	0.1239	0.0237	0.0408
6	7.0324	8.1809	104.1382	0.8596	0.7773	0.2227	0.2865	0.0548	0.0823
7	13.8820	60.7914	672.8639	0.2284	0.9293	0.0707	0.0761	0.0146	0.0261
8	3.4112	23.4199	11.9333	0.1457	0.9537	0.0463	0.0486	0.0093	0.0171
9	2.6350	9.1812	150.0807	0.2870	0.9127	0.0873	0.0957	0.0183	0.0323
10	8.5663	127.6164	492.6911	0.0671	0.9781	0.0219	0.0224	0.0043	0.0081
11	17.3210	260.5125	74.6104	0.0665	0.9783	0.0217	0.0222	0.0042	0.0080
12	2.7573	2.4846	9.1957	1.1098	0.7300	0.2700	0.3699	0.0708	0.0998
13	2.7614	12.8485	28.5192	0.2149	0.9331	0.0669	0.0716	0.0137	0.0247

Locations of trials: 1–Lisewo, 2–Kochcice, 3–Głubczyce, 4–Głębokie, 5–Krościna Mała, 6–Uhnin, 7–Głodowo, 8–Lubinicko, 9–Masłowice, 10–Jelenia Góra, 11–Rarwino, 12–Radostowo, 13–Lubliniec Nowy

5.2. Winter rye variety trials

The second series of trials consists of 15 experiments conducted in the 2004/2005 season with varieties of winter rye. Each trial was arranged in an affine resolvable design with $v = 18$ varieties allocated in $b = 12$ blocks grouped into $r = 4$ superblocks, each containing $s = 3$ blocks of size $k = 6$. The varieties were assigned to blocks in such a way that every pair of blocks from different superblocks had the same number of varieties, $k/s = k^2/v = 2$, in common. The average efficiency factor of the design was $\varepsilon = 0.8644$. The main results of analyzing the variety grain yields (in dt/ha) in the trials are presented in Table 3.

Table 3: Main results characterizing the individual trials conducted in 2004/2005 with varieties of winter rye

Loc.	s_1^2	s_2^2	s_3^2	s_1^2/s_2^2	$\hat{w}_1$	$\hat{w}_2$	$\hat{w}_2/\hat{w}_1$	$\hat{\zeta}$	$1-\hat{\sigma}^2_{\widetilde{\tau_i-\tau.}}/\hat{\sigma}^2_{\widehat{(\tau_i-\tau.)}_1}$
1	1.9425	6.7060	1.3438	0.2897	0.9119	0.0881	0.0966	0.0286	0.0336
2	5.6874	7.3419	50.7764	0.7746	0.7948	0.2052	0.2582	0.0766	0.0783
3	3.8871	12.9295	15.0177	0.3006	0.9089	0.0911	0.1002	0.0297	0.0348
4	4.4348	14.4064	251.0461	0.3078	0.9069	0.0931	0.1026	0.0304	0.0355
5	13.2458	56.8879	93.0674	0.2328	0.9280	0.0720	0.0776	0.0230	0.0275
6	19.5452	21.8576	103.4056	0.8942	0.7704	0.2296	0.2981	0.0884	0.0876
7	5.8929	16.5910	18.4494	0.3552	0.8941	0.1059	0.1184	0.0351	0.0404
8	9.0044	1.6846	10.8603	5.3452	0.3595	0.6405	1.7817	0.5283	0.2444
9	8.4979	14.8338	3.9750	0.5729	0.8397	0.1603	0.1910	0.0566	0.0612
10	13.0511	80.4549	8.0781	0.1622	0.9487	0.0513	0.0541	0.0160	0.0196
11	7.1101	8.9196	11.2786	0.7971	0.7901	0.2099	0.2657	0.0788	0.0801
12	13.2328	13.7123	21.4874	0.9650	0.7566	0.2434	0.3217	0.0954	0.0929
13	20.9710	41.5582	269.4008	0.5046	0.8560	0.1440	0.1682	0.0499	0.0549
14	16.1390	35.3568	80.1429	0.4565	0.8679	0.1321	0.1522	0.0451	0.0504
15	2.9391	9.0495	58.6720	0.3248	0.9023	0.0977	0.1083	0.0321	0.0373

Locations of trials: 1–Masłowice, 2–Przecław, 3–Dukla, 4–Ruska Wieś, 5–Seroczyn, 6–Kościelec, 7–Krościna Mała, 8–Uhnin, 9–Rarwino, 10–Lućmierz, 11–Lubinicko, 12–Kochcice, 13–Marianowo, 14–Pokój, 15–Jelenia Góra

5.3 Field pea variety trials

The third series of trials comprises 9 experiments conducted in the 2006 season with varieties of field pea. Each trial was arranged in an affine resolvable design of the same type as that used in the previous winter rye series of trials, i.e., with $v = 18$ varieties in $b = 12$ blocks grouped into $r = 4$ superblocks, each formed of $s = 3$ blocks of size $k = 6$. As previously, the assignment of varieties to blocks was such that every pair of blocks from different superblocks had the same number of varieties, 2, in common. The average efficiency factor was again $\varepsilon = 0.8644$. The main results of the analyses are presented in Table 4.

Table 4: Main results characterizing the individual trials conducted in 2006 with varieties of field pea

Loc.	s_1^2	s_2^2	s_3^2	s_1^2/s_2^2	$\hat{w}_1$	$\hat{w}_2$	$\hat{w}_2/\hat{w}_1$	$\hat{\zeta}$	$1-\hat{\sigma}^2_{\widetilde{\tau_i-\tau.}}/\hat{\sigma}^2_{(\widehat{\tau_i-\tau.})_1}$
1	5.8483	1.4444	8.5182	4.0491	0.4256	0.5744	1.3497	0.4002	0.2192
2	4.3043	3.3614	66.6738	1.2805	0.7009	0.2991	0.4268	0.1266	0.1141
3	2.3645	20.0832	32.2441	0.1177	0.9622	0.0378	0.0392	0.0116	0.0144
4	9.7544	81.5507	500.4608	0.1196	0.9617	0.0383	0.0399	0.0118	0.0146
5	3.8668	9.2882	146.2033	0.4163	0.8781	0.1219	0.1388	0.0411	0.0465
6	1.8209	2.2772	37.7074	0.7996	0.7895	0.2105	0.2665	0.0790	0.0803
7	3.0899	38.9040	5.5707	0.0794	0.9742	0.0258	0.0265	0.0079	0.0098
8	5.6894	6.9968	21.2202	0.8131	0.7868	0.2132	0.2710	0.0804	0.0814
9	14.9739	58.9732	63.2182	0.2539	0.9220	0.0780	0.0846	0.0251	0.0298

Locations of trials: 1–Głębokie, 2–Kochcice, 3–Pawłowice, 4–Rychliki, 5–Karżniczka, 6–Kawęczyn, 7–Zybiszów, 8–Czesławice, 9–Rarwino

6. Concluding remarks

- As can be seen from the presented methods and their applications, the affine resolvable block designs are advantageous not only because of their desirable optimality properties, but also because of their simplicity in the analysis of experimental data.

- As to the first properties, Bailey et al. (1995, Section 3) have shown, referring to the analysis under the intra-block submodel, that in the class of resolvable designs the affine resolvable designs are optimal with respect to many criteria, including A-, D- and E-optimality.

- These optimal properties of an affine resolvable design are preserved when going from the intra-block analysis to the analysis combining the intra-block and the inter-block-intra-superblock information, provided that the grouping of experimental units (plots) into blocks is sufficiently successful for increasing the ratio σ_2^2/σ_1^2 over 1 (i.e., decreasing σ_1^2/σ_2^2 below 1). (See also Caliński and Kageyama, 2003, Lemma 7.1.2 and Theorem 7.1.1, and the discussion following them.)

- The simplicity of the analysis of any experiment conducted in an affine resolvable block design should, additionally, encourage researches to the use of this type of designs when dealing with a large number of treatments (varieties). This has already been explored by some institutions responsible for evaluation and registration of new agricultural crop varieties.

- Results from the analyses of three series of variety trials have shown that in the majority of cases the grouping of plots into blocks has been successful.

- Furthermore, when there are some deviations from the basic principles of experimental design, from the principle of local control in particular, the combined analysis helps to arrive at sufficiently precise inferences. This can be achieved avoiding any numerical difficulties, such as those connected with iterative procedures usually involved in analyses of experiments in other than affine resolvable designs (see, e.g., Williams, 1977).

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