Spatial Economy Classics
4

Ryszard Domański

Complexes of transport networks

Bogucki
WYDAWNICTWO
NAUKOWE

Ryszard Domański

Complexes of transport networks

Spatial Economy Classics

4

Ryszard Domański

Complexes of transport networks

Bogucki Wydawnictwo Naukowe • Poznań 2021

UNIWERSYTET
EKONOMICZNY
W POZNANIU

This work has been published in English on the initiative and with the financial support of Professor Maciej Żukowski, Rector of the Poznań University of Economics and Business, as part of celebrating 95 years of the Poznań University of Economics and Business (1926–2021).

Translated by Ewa Dratwa

ISBN 978-83-7986-353-2

Bogucki Wydawnictwo Naukowe
ul. Górna Wilda 90, 61-576 Poznań, Poland
phone +48 61 833 65 80
e-mail: bogucki@bogucki.com.pl
www.bogucki.com.pl

Printed in Poland by Uni-Druk, Luboń

Contents

Foreword 7
Preface 9
I. Introduction 13
  1. The issue 13
  2. Literature review 16
  3. The method 25
II. Transport network models. The notion of a complex of transport networks 29
III. Approximation 49
  1. The properties and functions of various types of transport. Allocation premises 49
  2. Elementary complexes 65
  3. Evolution of complexes of transport networks 70
  4. Differentiation of geographic and economic space 79
IV. A complex of transport networks: a model 93
  1. The travelling salesman model 94
  2. The copper-centre model 95
  3. The centralised network model 95
  4. The anisotropic model 97
V. The typology of complexes of transport networks 109
VI. Verification 113
Bibliography 125
Subject index 131
Summary 135
List of Figures 139

Foreword

Since time immemorial, science has developed owing to the brilliance and talents of great thinkers. Each discipline of science can boast great minds.

Regional Science, initiated in the 1950s by the memorable Walter Isard (1919–2010), gradually spread from Pennsylvania to the entire globe. In Poland, then a country located behind the Iron Curtain, Professor Ryszard Domański acted as a pioneer of Regional Science. The methodological techniques employed in his works and the academic level of explaining the processes at hand definitely differed from the other Polish publications on economic geography, later on labelled socio-economic geography.

As part of the "Spatial Economy Classic" series, the Spatial Economy Society has published an English version of Professor Ryszard Domański's habilitation dissertation on "Complexes of Transport Networks" from 1963. This work only confirms the viability of the research carried out at that time by Professor Domański. It is also substantial evidence of his connection with the modern methodological trend within the quantitative paradigm. All his academic life, Professor Domański has been faithful to this vision of pursuing "Regional Science".

It is our pleasure to present to you this classical work of an international community.

Waldemar Ratajczak
SES Chairman

Preface

It is a great honour for me to write a small preface for this book, celebrating the scientific heritage of Professor Ryszard Domański. But it is also a great personal pleasure for me to express my feelings of deep appreciation for a great scholar and a great person of humanity.

Professor Ryszard Domański has a long standing history as a bastion of regional science and economic geography. For decades he has been a leading academician in Poland whose works were always highly recognized as real scientific quality. He has published numerous contributions in the Polish language, but he was also clearly present with his writings in the international literature. And he participated in many international and European conferences. Wherever he gave a scientific presentation or exposition, the participants were always impressed by this strict logic, transparency and clear argumentation.

His multitude of contributions are generally characterised by a rigorous scientific structure: a solid theoretical foundation, a crystal-clear analysis, and an original and innovative approach. He was able to link emerging issues (e.g. the relationship between Christallerian central place theory and the complex space-economy) to new insights from systems theory and complexity analysis. In this way he has been of great influence on scholars in Poland, but also elsewhere.

Personally, I have been one of the privileged young persons to have met Ryszard Domański already in the 1970s. His friendly and open character, his interest in unconventional approaches, and his sense of a strict scientific habitus made a deep impression on me. And we were able to keep good contacts over all these years. He was for me one of the leading masters who taught me to follow strict scientific rules in academic research. And I am really grateful to him for all his seminal contributions to regional science.

One of his early interests in regional science can be found in this habilitation study on ‘Complexes of Transport Networks’. This early but still very relevant study demonstrates the scholarly talent of Professor Domański. It is not an easy study, but he manages to formalise the complexity of the space-economy in the context of a general hierarchical spatial system inspired by the seminal works of Christaller and Lösch. In this study he shows already the principles of spatial complexity theory, an analytical approach that has become rather popular in the past decades. So he was a predecessor of the formal and quantitative direction in regional science and economic geography. He left many traces in the field of spatial science; even contemporary works on space syntax and urban morphology can be positioned in the rich science history developed by Domański. This book on the complexity of transport networks which is now available for a large international audience illustrates and documents the scientific calibre of Professor Ryszard Domański. Its merits transcend by far the borders of Poland; it addresses clearly a worldwide audience.

Peter Nijkamp

INSTYTUT GEOGRAFII
POLSKIEJ AKADEMII NAUK

PRACE GEOGRAFICZNE NR 41

RYSZARD DOMAŃSKI

ZESPOŁY SIECI KOMUNIKACYJNYCH

PAŃSTWOWE WYDAWNICTWO NAUKOWE
WARSZAWA 1963

I. Introduction

1. The issue

The development of transport leads to increasingly more diverse forms. Each of them performs its specific, unique functions, at the same time supplementing the other forms. Whenever the division of functions is not unambiguous, one form can replace another. Therefore, numerous interdependencies are established between the forms which tend to change and complicate as a result of the uneven rate of the development of the various forms of transport. Sometimes in the process hypertrophy accompanies the most dynamic forms or occurs as a result of new forms severing the old global ties. In the new transport conditions, new ties are created. Therefore, simultaneously with the process of differentiating functions, transport is submitted, so to say, to integrals, resulting in transport formations with complexes of transport networks as their most robust elements.

The author assumes that the formation of transport networks is a correct process which should be examined in order to identify the laws governing them [59]. Geography of transport is predominantly interested in the laws of the spatial structure of formations. While they are a part of the coexistence laws, they can be brought down to causal laws i.e. regular relations between the conditions of the formation of complexes and their spatial structure.

Following the examination of functions and correlations, we need to look for laws which subordinate the spatial structure to indispensable and sufficient conditions. The premises should include economisation of transport activity interpreted as the praxeology ideology. In the conditions of proportionality of the goal and the measures, the directive assumes the form of the rule of economy which represents the efforts to obtain the best possible effect with specific funding volumes or the smallest funding volumes for achieving a specific effect.

The results of this search should ,make it possible to create a theoretical model of a unit of transport networks. While systematic transports sciences have failed to create it so far, it has been of interest to economic geography. For this reason, discussing it in geography and transport works is definitely justified and desirable. Of course one model can contain only the most general conditions where the laws of the spatial structure of complexes are true and mutually related. When confronted with reality, this model tends to be very diverse. It is possible to provide more details by means of typological diversification.

To this end, actual complexes of transport networks from various countries need to be examined with attention paid to the similarities of the lines and nodes, their locations, functions and development rather than their unique nature. This scientific approach allows to establish order in their magnitude and to explain the congruence of the spatial structure together with the conditions of establishing and developing the most characteristic complexes. As a result, the types of complexes need to be distinguished. The ultimate results may also contribute (in an extent typical of transport geography) to the development of notions related to the system of economic regions and their typology.

Finally, diversified and brought nearer to reality theoretical complexes may be verified at the final stage of the research procedure.

In the face of incomplete factual and analytical materials, in the initial studies dedicated to complexes of transport networks it is difficult to identify their characteristic universal dependencies. Without abstaining from identifying them (which needs to be deemed as the goal of a collective effort), we should start from identifying historical and fragmentary generalisations. This work is an attempt at that.

The forms and formation of complexes of transport networks (morphology)[1] are typically described by mans of historical, individual sen-

---

[1] The term has not yet been used in literature on transport. By applying it, the author related it closer to geomorphology than settlement geography.

tences (basic, observation sentences) without strictly general sentences or laws. It is a rare case to even detect loose pertinences in the repetitions or more frequent coincidences of phenomena coexisting in a specific time and place. This condition is attributed to strong individualisation of the complexes and the great number of their unique features.

However, in this respect transport geography is not in a unique situation. Numerous sciences have the goal of identifying stochastic laws i.e. ones manifesting themselves only when a special type of events repeats itself on a mass scale. The studies are awarded with valuable theoretical results. This is possible because the research methods have been enhanced and new ones have emerged, better tailored to the nature of the phenomena in question. Some geographic and transport works should follow suit.

It seems superfluous to put forward arguments for the nomothetic approach and against the exclusivity of the idiographic approach in transport geography. Similarly, there is no need to prove that descriptive as well as theoretical consideration of separate areas (aspects) of economy in space is desirable (with complexes of transport as a special case). An opposite conclusion does not result from the harsh criticism of the theories of the settlement pattern, especially that of W. Christaller, for their formal nature and abstraction based on premises too cut off from reality.

It is obvious that in the course of regarding partial issues we need methodological self-knowledge which allows to combine them properly with the whole as they occur in reality. To better understand the relations, the coordinates of the issue in question need to be established on a map of geographic terms or a grid of logical places as a logician would put it. The logical location of complexes of transport networks is as follows:

1. In an issue-related approach, complexes of transport networks are a node-like component of a regional structure while the unit theory is a component of the theory of economic regions;
2. In a systematic approach, complexes of transport networks are among the major issues of transport geography. This is confirmed by authors

---

Sometimes, in criticism of works on settlement geography, morphology is identified with describing visible features of settlements. From the point of view of semasiology, it is incorrect. What is justly criticised is not going beyond morphology.

most in favour of the regional (comprehensive) direction of economic geography[2]. What is more, complexes are examined as part of other disciplines of science: economic, historical and technical. However, disregarding the different views on the disciplines, viewing complexes as part of economic geography seems to be most promising as the discipline revolves around examining the structures of complex phenomena.

On top of cognitive goals, this work has also practical goals, namely expanding the scientific basis of coordinating the routes of various types of transport on a regional scale. While there is a need and possibility of coordination in the scope of the specific types of transport, it is the various types of them where the postulate to negotiate the components is truly important. Theoretical inquiries may serve as the solution, identifying the assumptions, operations and results of an optimal complexes of transport networks. Thus it is possible to evaluate the value of the existing complexes and to foresee their future development. Transport policy should strive to introduce the causes which, based on the principle of a determined succession of events, trigger off optimum complexes as the effect. Regional planning is directly affected by the ways in which such complexes can be derived from specific regional conditions.

2. Literature review

Literature on the connections between geography and transport presents few issues that would be as obvious and, at the same time, so far from a proper solution, as transport complexes. As far as I know, there is no literature on the subject that would treat transport complexes as a discipline in itself. At best, works dedicated to related issues present loose thoughts, typically not exceeding most general statements. If research into complexes is mentioned directly, it is usually in the form of a postulate, followed by separate characteristics of the specific types of transport. Genuine complex-oriented research is non-existent.

---

[2] According to J. Sauchkin [93], transport geography deals with „… distribution and connection of various types of transport and the transport network".

This is not an isolated opinion. It is reflected in E.L. Ullman's words [103]: "In American geographic literature there are hardly any studies on the relations between the various forms of transport". This statement may be deemed representative of the world transport geography. Similarly, economic sciences are not sufficiently interested in the subject while there is impressive literature on competition and cooperation of various types of transport. For example, W. Linden [62]wrote: "The change in the transport structure which occurred in the past 30 years, is gaining great importance yet it needs to be said – lamentably – that it is not sufficiently taken into account in traditional transport science as well as the official transport policy". However, a notable exception should be mentioned, namely establishing several years ago the Institute of Complex Transport Issues at the USSR Academy of Sciences. The Institute provides an excellent organization and technical platform for research into transport as a whole. The first publications suggest that the research first focused on coordinating transport by various means.

Complexes of transport networks are a typically comprehensive issue. It implies that the specific types of transport should be viewed as parts of a single item but also that this item should be examined from different angles. I strive to consider the point of view of geographic and economic and, partly, technical sciences. Accordingly, I resort primarily to geographic, economic and technical literature.

It is not my goal to review historically works on the subject.[3] To me, it is more important to capture in the existing literature thoughts of importance to the subsequent research into the subject. In the process, gaps in the source materials will also be identified. The literature relatively closest to the subject has been grouped in the four subsequent chapters.

2.1. Geographic descriptions of transport

They refer to areas of different sizes: the entire globe, the continents, countries, zones, districts, cities and villages. Transport is presented

---

[3] The bibliography of works on transport geography up to the 1920s is included in the book by K. Hassert [39]. More recent literature in German, English, French, Russian and Polish is listed in works by E. Otremba [80], E.L. Ullman [103], S. Berezowski [7], W. Nikolski [78] and Z. Chojnicki [18].

from the point of view of regions or branches. Initially, these were simple descriptions limited to formal features and utilitarian information (for trade) [3], followed by descriptions explaining the phenomena of transport – first geographic causes (determinism) and later on also non-geographic causes. This type of works provide factual materials on transport lines, nodes and regions of various levels. Their value to the subject in question depends on the degree to which the cause is explained as well as the details and completeness of the information.

From this point of view, works dedicated to the so-called general transport geography are of limited value. They prove useful only at the initial stage of the research procedure when it comes to a preliminary overview of the specific branches and regions of transport. On top of J.G. Kohl's work [49] in which the geometric structures of the transport network are more significant than geographic descriptions, the following authors should be named: F. Richthofen [90], K. Hassert [39], E. Friedrich and W. Schmidt [33], A. Hettner [41], E. Cleef [21], R. Capot-Rey [14] and E. Otremba [80]. Some of these authors like A. Hettner, R. Capot-Rey and E. Otremba as well as K. Sapper [92] attempted at classifying transport in the world. However, they failed to avoid simplifications and logical inconsistency in their attempts at identifying multi-dimensional units. S. Berezowski [8] drew the classification of continental transport systems.

More detailed information is provided in regional monographs typically including specific countries or smaller administrative units. This holds true for special transport monographs as well as general geographic and economic monographs with chapters dedicated to transport. Both types of work are represented by a very large number of publications. Authors of special monographs include for example H. Lartilleux [54], J. St. John [45], R.G. Lewis [61], K. Remy [89], A.L. Batalow [5], I.W. Nikolski [78], T.S. Khachaturov (ed.)[16], S. Leszczycki [58], A Wrzosek [108], S. Berezowski [6]. There are also numerous monographs of the specific transport lines and nodes (e.g. the Trans-Siberian Railway, the Pan-American Highway, the trans-Sahara routes, Volga-Don Canal, the Saint Lawrence Seaway).

The traditional issues of the impact of the geographic environment on transport, largely covered in most of the mentioned works, have been also included in separate publications. P.H. Schmidt [42] discusses it in a general way while O. Blum [11] from the point of view of technology, dedicating most attention to the impact of terrain on the construction of transport lines and the development of transport networks. The mutual influence of the geographic environment and transport is analysed by F. Barciński [4]. Genetic explanations of the contemporary transport networks have been included into geographic-historical and historical works.

As is typical of source materials, the literature of the first chapter represents a contradiction between the detailed nature of data and its completeness. The works presenting all types of transport are not sufficiently detailed even if they relate to low order regions. The detailed works are incomplete, not only in themselves (they typically focus on one means of transport) but also because, even if they refer to the same territory (which is rare), the entire transport is hard to reproduce. Therefore there are no publications that would provide sufficient factual materials for research into complexes of transport networks.

2.2. Transport in the theories of production location

Literature of this chapter is rather loosely connected with the subject in question. Indeed, it pertains to completely different issues. A study into the literature brings about two benefits: 1. It allows to trace mathematic methods used in spatial research; 2. By explaining, more or less adequately, the location of single production plants, it is conducive to a better understanding of the economy's spatial arrangement and indirectly of the arrangement of transport networks.

The most important works are the ones whose authors explain the location-related importance of various means of transport. Unfortunately, they are rare. J.H. Thűnen's [100] plains are not crossed by any large river or canal. Transport is arranged by horse on roads. A very well developed network of these roads ensures equal accessibility while the costs of transport are proportionate to the size and importance of the cargo. The uniformity of transport lies also in the concept of W. Launhardt [55]. A. Weber [106] tries to take a step further, beyond a simple case of the ratio between the cost of transport, the weight and the distance. He introduces to his theory complications in order to bring it closer to reality; among other things, he assumes the existence of different types of transport and various tariffs. He avoids the complication in an interesting way; however, it cannot be applied to complexes of transport networks – namely, by calculating actual weight and distances into "ideal" ones. It is assumed that cargo from higher tariffs is heavier and the routes of more expensive transport are longer and vice versa.

Critics of A. Weber together with L. Bortkiewicz, W. Sombart, O. Engländer, A. Predöhl, H. Ritschl and H. Weigmann do not contribute much to the subject in question, the most interesting being the location theory of A. Predöhl [88]. The rule of substitution on which the theory is based manifests itself, according to the author, in the arrangement of transport costs. Namely, when the production location is shifted, some transport costs are substituted by others, e.g. the costs of transporting raw materials grow while the costs of transporting finished goods decrease. In order to maintain balance i.e. the minimum of the total transport costs, an increase in one area should be compensated with a decrease in another area. These findings lead to an observation that the rule of substitution also relates to the spatial arrangement of various types of transport. However, this is a separate issue that calls for a solution. It is not provided in the balance theories of L. Walras, V. Paret or G. Cassel to whom A. Predöhl refers. This is because they apply only to an area where transport costs equal zero, perfectly active capital and labour and equal production conditions everywhere i.e. they apply to an area of a one-point market.

It was T. Palander [82] who actually introduced to the theory of location the issue of mixed transport and who examined the resulting deformation of isodapanes and a shift in the minimal costs of transport. Palander also presents his findings about the dependencies which H. Stackelberg [99] later on coined as the law of transport collapse. E.M. Hoover [42] also draws attention to the location implications of developing new types of transport of different cost and tariff structures. They are an object of interest of W. Isard [43]. On the other hand, A. Lösch [64] was critical about solving the problem of location based on the transport factor and did not elaborate on this orientation. On the contrary, in his critique of Weber's theory and more recent theories, he concluded that if variability of demand is considered, the search for minimal costs of transport is senseless. His attention shifts towards the spatial arrangement of the entire economy and therefore he touches upon, albeit superficially, the general arrangement of a complex of transport networks. E.D. Khanoukov [17] wrote a valuable work on the importance of transport in distributing socialist production.

2.3. Theories of transport networks

While works on this subject are relatively closest to the issue of transport networks, they do not exhaust the subject. What needs a solution is the issue of the diversity and the relations between various types of transport in complexes. The authors typically avoided this huge complication, frequently limiting themselves to looking for optimum forms of networks for transport in general. When we assume that specific forms relate to vehicle roads or to railroads or waterways (*toutes proportions gardées*) we can design a scheme of diverse transport. However, it may be barely a starting point for considerations of complexes of transport networks. Neither of the schemes take into account the issues of the network's dynamics or the diversity of geographic and economic space.

The search for optimum forms of transport networks was commenced in the ancient times in city planning. Plato (*Laws*) and Vitruvius (*On Architecture*) considered the concept of an ideal city and established the rules of a concentric arrangement of roads which has survived to our times through the theoreticians of the Renaissance and the theoreticians of war engineer art of the 17th and the 18th centuries reached its peak of popularity in the 19th century [27].

J.G. Kohl was the first geographer to make an attempt at arrive at a theory of transport network [49]. However, initially there was no one to continue his work. What is more, he was heavily criticised by F. Ratzel and K. Hassert, to name a few, who accused him of being schematic and removed from reality. While there was a grain of truth in this criticism, there were also positive elements in the theory. A. Hettner and O. Schlűtter were the only eminent geographers to support J.G. Kohl's concept but failed to develop it.

E. Sax, a well-known transport theoretician, made an attempt at an economic interpretation of transport directions [94]. He never constructed a network model but he rightly viewed a number of cases of emerging transport directions and went a step further than Kohl in adopting the direction law of traffic (*das Richtungsgesetz des Verkehrs*).

The major part of the transport network theory was considered within urbanism, transport engineering and city planning. Theoretical considerations intertwined with the practical needs of public urban transport. Of significance are attempts at modifying a chessboard-like grid of streets, the oldest geometric grid used for example in Roman colonies and, on a large scale, in North America. The same holds true for the development of the hexagonal grid theory, the best motivated geometrical layout developed in the form of the trigonal layout. The significant authors include W.H. McLean [71], C. Kehr [48], A.C. Comey [22] and H.L. Sierks [97].

Geographic literature from the 1930s is notable for two works on the theory of the transport network. H. Haufe [40] evaluated the properties of various geometrical figures against two postulates: 1. The biggest value of the area/circumference ratio, 2. Covering in entirety the spherical surface of the Earth with a regular network. He concluded that only a regular hexagon fulfils both postulates therefore it is the best model of a transport network. The other author whose work was evaluated in surprisingly contrary ways (ranging from enthusiastic to very critical) is W. Christaller [20], [28]. In his pure theory of settlement advanced from an analysis of the reach of central goods and services assuming that all the parts of the settled area will be supplied by the smallest possible number of central estates (places), the distribution of these estates is subjected to geometrical laws and forms a hexagonal network. Anomalies in an ideal system based on the rule of supply trigger off two other rules: that of transport and administration. According to the rule of transport, the distribution of central estates is best when the biggest possible number of important estates are located along the possibly straightest and least expensive transport line connecting two large cities while less important estates are kept aside.

A concept of the transport network close to that of Christaller was developed by A. Lösch, a theoretician of spatial economy [64]. In his simplified model of an economic region, the system of transport lines and points stems from an ideal arrangement of the market areas. It is complicated by natural obstacles and the fact that transport points affect the emergence of market areas. W. Isard [43] introduced to A. Lösch's geometrical constructions another complication: an uneven distribution of people. However, his modified model serves to draw attention to the change to the market's size and form, at the same time maintaining the transport system, limited to six major lines, unchanged with a single exception.

In Polish literature on the subject, K. Dziewoński [27] was the first to provide a synthetic approach of the theory of a transport network and shed light to complex issues. After WWII a number of articles were written predominantly by transport engineers and urban planners.[4] However, their scope of interest does not exceed the city and the suburban area. The authors included, among others, T. Baniewicz, M. Barbacki, B. Brukalska, Z. Lilpop, S. Plewako, L. Tomaszewski and P. Zaremba. The works of Z. Wasiutyński should be treated separately. The idea of a geometrical arrangement of roads has been most fully justified in these works. Many years of studies have been completed by the recent in-depth research into Designing transport arrangements [104] whose author presents extreme transport arrangements (roads and mass communication) by resorting to geometry and mathematical analysis.

2.4. Transport coordination

Coordination of various means of transport is one of the major problems of the contemporary transport policy. Extensive literature on the subject has been written all over the world; with respect of the volume, only literature in chapter one can compare. The authors wonder how to eliminate harmful competition and to coordinate various types of transport. However, as economic theory indicates and experience confirms, in capitalism this problem cannot be fully solved while the suggested preventive measures (related to organization, tariffs, licences, taxes, subsidies) are of merely palliative value. This type of measures tends to be short-lived and their suitability – poor.

---

[4] Most of them were published in "Miasto" magazine.

Much more suitable are works which, in search of the bases of transport coordination, analyse the technical and economic properties and therefore identify the range of the rational actions on the part of every type of transport. Examples of this research procedure can be found in general works dedicated to transport economics as well as specialist works focuses exclusively on transport competition and cooperation. The first group includes, among other things, works by C. Pirath [85], D.Ph. Locklin [63] and T.S. Khachaturov [15]. The latest publications from the second group include works of J.R. Meyer, M.J. Peck, J. Stenason, Ch. Zwick [72], H. Watermann [105], K.P. Böhm [12], W.W. Zvonkov [109]. I.I. Belousov and A.W. Komarov (ed.)[9] and Z. Patalas [83]. Polish literature is represented by, among others, M. Łopuszański [66], [67], Cz. Michalski[5] and W. Młodecki [73]. Lately, large-scale and comprehensive research has been started into the spatial structure of freight transport [107].

The author makes numerous references to the technical and economic characteristics of the specific types of transport. In particular, he makes use of an analysis of the relevant spatial properties of each of the types. However, in this analysis he failed to present sufficiently the mutual spatial relations and therefore the subject is waiting for expansion.

Works presenting mathematical methods in the search for the minimal loads (transport costs, time of transport) represent a new chapter on rationalization of transport. There are various models and algorithms of solving the so-called problem transport in applied mathematics (linear programming), by A.N. Tolstoy [102], L.W. Kantorowicz [46], [47], T.C. Koopmans [51], G.B. Dantzig [24] to mention only the most famous ones. Hoever, neither of them solves the problem of transport diversified with respect to the type; it is therefore hardly surprising that there is no reference to the problem of complexes of transport networks. The suggested solutions can at best serve to solve simple cases in auxiliary complexes. More general conclusions on complexes of transport networks[6] related to

---

[5] Articles in „Transport" and „Przegląd Kolejowy" (1956 and subsequent editions) on the comparative effectiveness of local railways and car transport.

[6] Attention should be paid to works of W.L. Garrison [36], [37] where the author applies methods of linear programming and variance analysis. However, it pertains only to one type of transport road network, namely railroads.

complexes of transport networks will be drawn following adaption (combined with modification) of the methods suggested now to more complicated issues and application of new methods (tensors and variants are particularly promising).

In the search of inspiration and associations, I have also resorted to literature rather far from the work's subject, for example literature on praxeology, biology and physics.

3. The method

Elementary complexes of transport networks are the major element of the concept presented in this work. I define them as simple but regularly repeating roads of various types of transport which can mutually supplement one another (complementary nature) or substitute one another (substitution). The structure of these connections stems mainly from the requirements of transport itself (functional) but is disrupted as a result of the diversity of geographic and economic space as well as historical conditions.

These complications were originally taken into consideration only to the extent to which they affect the shape and structure of complexes of transport networks; the idea was to build a model of a complex. Later on, as more details were provided on the theoretical model, the types of complexes were identified further on subjected to zoning and verification with respect to invariable features.

At the specific stages of the research procedure, the relevant conclusion methods were applied, parts of the comparative observation method which is the basic method in geography. Therefore, in the examination of elementary complexes and the structure of the model the induction method was applied predominantly but not exclusively. In identifying the types of complexes, it was the deduction method and in the course of verification – the reduction method.

These general methods assume in the work specific forms enforced by geographic and economic sciences. The methods of describing, identifying and confirming spatial relations, more specialised in comparison with logic, have been taken from mathematics (tensors, calculus).

Mathematical concluding was applied when in a set of not inconsistent axioms which are premises some variables appeared (transport distance, transport costs). Praxeological conduct and, first and foremost, the rule of economy, were sometimes deemed axiomatic. These rules lead to deducing useful generalisations which are even more powerful with respect to logical imperative than the generalisations resulting from induction which also necessitate numerous observations and complicated historical and statistical analyses. The rule of economy is helpful, especially in identifying the optimal shapes of a complex of transport networks which may be regarded a special case of optimal use of means, not identified as a result of marginal analysis and linear programming.

The map is the fundamental tool for examining complexes of transport networks. Mathematical methods do not eliminate the map nor do they play down its significance. However, they require projection of increasingly complicated phenomena in an increasingly precise way. Graphic representations facilitate interpretation of logical and mathematical conclusions; without a map it would be difficult to verify any conceivable theory of complexes of transport networks.

A specific methodological attitude stems from a historical scope of the laws of the spatial structure of complexes of transport networks. Only some transport laws are universal by nature in the sense that they operate on all the levels of transport development. These are the laws of covering distances and they remain valid with respect to complexes. However, complexes are typically accompanied by laws of their spatial structure which are in force only in specific transport conditions, namely in conditions of a specific set (complex) of various means of transport. These conditions change as new means of transport appear and old ones slip into oblivion; in a new set (complex), new laws appear. Right now, of special importance to science and practice is research into changes to the transport conditions and new laws resulting from the expansion of car transport. Finally, the development of complexes is affected by economic laws (which have a historical range), specific to social formations, economic laws shared by all various social formations and the laws of the superstructure [53]. This is because complexes always develop in specific social and economic conditions which form the basis for the laws to operate. These laws permeate all the areas of social and economic life including transport, modifying the laws of covering distances and the laws of the spatial structure of complexes of transport networks.

Therefore, a historical perspective coupled with a functional aspect should be supplemented with a spatial perspective. This postulate is important from the point of view of the work's cognitive goal including not only the relations of coexistence in space but also the causality.

In order to obtain more complete results in a specified scope, there was a need to highlight the spatial dimension of the issue, limiting the time and function aspects. As a result, issues like characteristics of the various types of transport and evolution of complexes of transport networks have been only presented in a scope necessary to generalise and verify. One could say that they have been treated not systematically but pragmatically.

An implicit assumptions of the theoretical premises of complexes of transport networks is rational cooperation of various types of transport. In fact, this cooperation is possible only in the conditions of social property of the basic means of production including means of transport. Therefore, theoretical complexes will have the biggest number of features common with complexes of transport networks affected by a socialist economy. Complexes affected by capitalism have many deformations which cannot be attributed to actual transport needs. This makes the complicated induction and verification even more complex.

II. Transport network models. The notion of a complex of transport networks

The presentation of the transport network models can be started with the model developed by J.G. Kohl. His goal is to explain the influence of various forms of terrain[7] on transport and the resulting settlements. He concludes that with respect to their impact on transport, the various forms of earth surface are not comparable. Hence the postulate to conduct a separate examination of the impact of the specific forms. However, despite all the diversity, each form (however irregular) becomes increasingly similar and can be compared to this or other geometric figure. The characteristics of these figures should be examined together with the properties of transport against these figures in order to identify any general laws.

According to Kohl, the circle is the most perfect figure and therefor the other ones are less perfect the more they deviate from the circle. On top of the circle, Kohl considers the square, the equilateral triangle and the ellipse. At first he analyses each figure separately, assuming identical conditions for transport and settlement. He poses a question about the conditions in which internal, external, transit and cabotage transport is provided. Figure 1 presents the graphic solutions.

The shapes of a transport network result from two different trends: the trend of every settlement site to connect with the other sites with a set of straight lines connecting all the sites representing it in a geometric way and a trend for many directions channelling into one. The condition for that is that transport may only take place along technically prepared

---

[7] His understanding of terrain is close to the contemporary understanding of the surface of the earth.

roads involving cost-consuming investments. Some directions prove particularly profitable, attracting a large number of carriages and therefore their technical equipment is constantly taken care of. This is how main roads emerge with side and access roads branching out from them. The status of the transport roads is reflected in the status of the settlements located along them.

Having analysed partial transport arrangements in isolation, Kohl wonders how the transport and settlement networks are shaped when these elements are combined into a single general arrangement. A lot depends on the fact which of the partial arrangements develops as first because the first transport lines and settlements partly condition the location and the development of the subsequent ones. The assumption should be that internal transport develops as first as humans tend to move around most easily in their own environment. In comparison with internal transport, cabotage transport reflects altogether different trends: one is concentrically-oriented with its influence wanes towards the circumference. Another trend is about clusters forming along the circumference with transport's influence waning towards the centre. There is a similarity: in both scenarios, traffic on radial roads develops. We may assume that in similar conditions, separate roads will not be created for both types of transport; rather, shared radial roads will be built. Originally there are four of them, later on eight, then if need be – sixteen etc. but they will always converge at the same angle. At the end of the radial roads, cabotage ports will grow which will not, however, contribute to the importance of the roads themselves as long as the freight remains unchanged. The significance of the centre of enclave will diminish as a result of the centrifugal forces on the circumference. The emergence of external and transit transport does not change significantly the arrangement of the transport network. An increase in freight affects only the number of transport operations as well as the number and size of the settlements.

Adopting this line of thinking, Kohl finally reviews the deviations in a complex of transport and settlement networks resulting from the co-existence of various figures. Nevertheless, the entire structure is way too simplified and formalised (not to mention geographic determinism, criticised so many times).

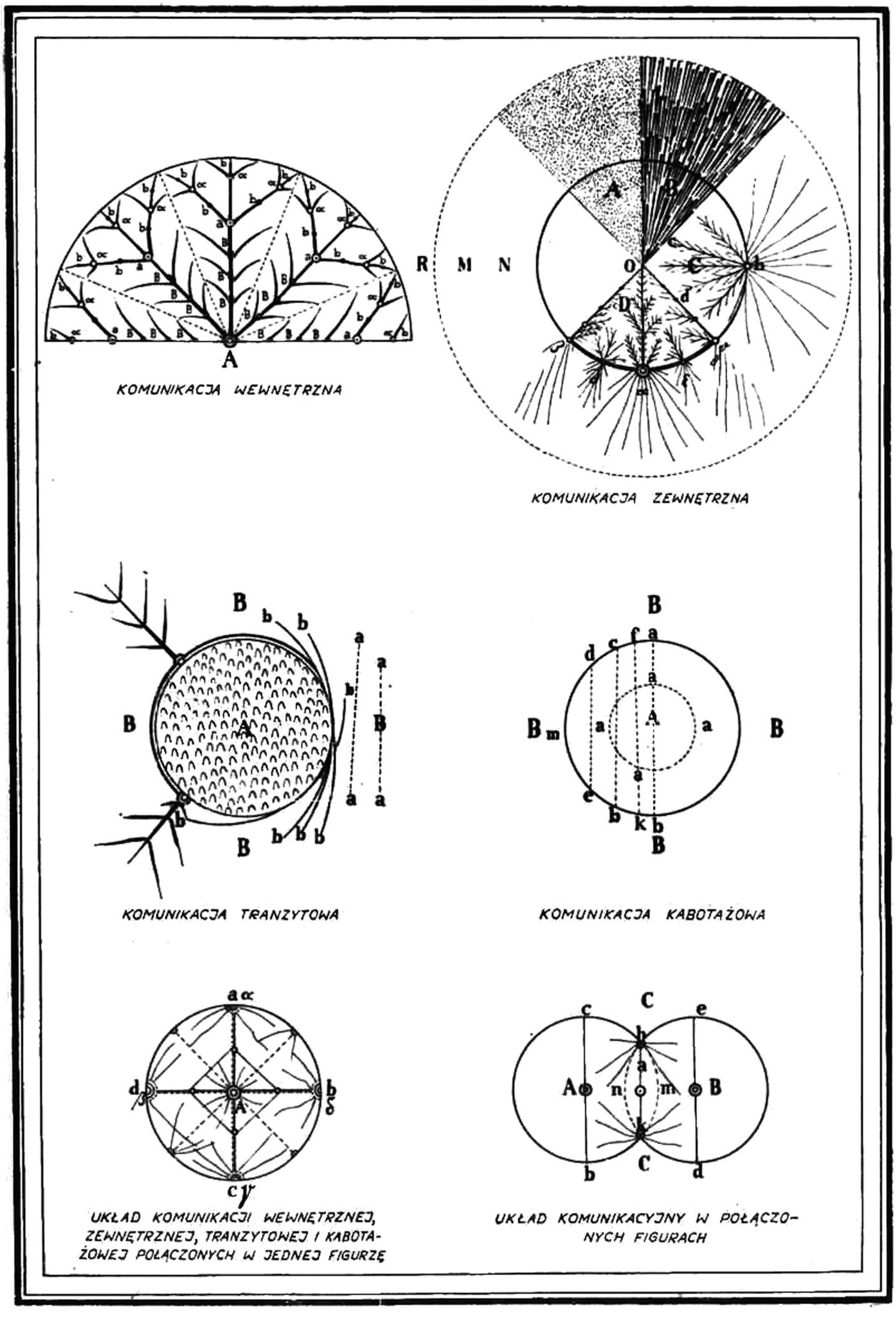


Fig. 1. Transport network in J.G. Kohl's figures

W.H. McLean built a model of a chessboard-based transport network with special purposes for the colonies (a “grid system”). According to the author, the network should connect the inside of the colony with the sea in the simplest possible way, taking into account the economic radiuses of use typical of the different types of land and sea transport. The starting elements are sea ports whose distribution (300–500 miles from each other) stems from an interregional plan. A specific region has a port at its disposal to which rectangular land transport leads. The arrangement’s axis is a railway running from the port into the land. The axis is connected at right angles with side railways or main (macadam) roads parallel to each other at a distance of 100–150 miles. They play the role of support lines while their crossings mark locations where cities are formed. Every 50–75 miles, beaten roads of secondary importance run at right angles to these lines or, possibly, natural paths along which vehicles can ride. At the crossroads, rural settlements emerge (Fig. 2).

At present, reconstruction of the transport layouts of the former colonies is a burning question. However, McLean’s model does not provide

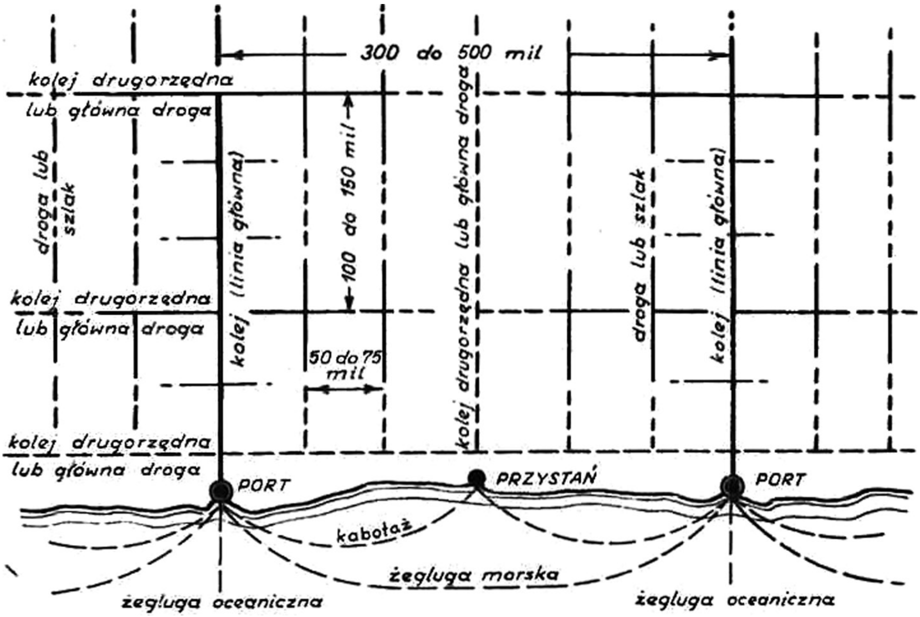


Fig. 2. Design of a transport network for a colony. An interregional arrangement. Based on W.H. McLean’s model

it with an appropriate basis. This is because the model is unilaterally sea-oriented (oriented to the outside) and fails to appreciate internal relations. Obviously, the author assumed the colonies' economic dependence on the metropolises and maintaining raw materials economy in the colonies which is unacceptable, especially in prospective projects which are indispensable in shaping transport networks. Developing a new transport network model that would consider the national interests of the former colonies is a graceful task of great importance.

Obviously, the author assumed a colony's economic dependence on the metropolis and maintenance of raw materials policy which is unacceptable, especially in future projects which are indispensable to the emergence of transport complexes. Development of a new model of a transport network taking into account the specific countries' national interests (the former colonies) is an enviable and most important task.

The latest models of a transport network are typically based on a hexagonal arrangement of connections morphing into a trigonal system. Many theoreticians arrived at this system almost simultaneously. As for

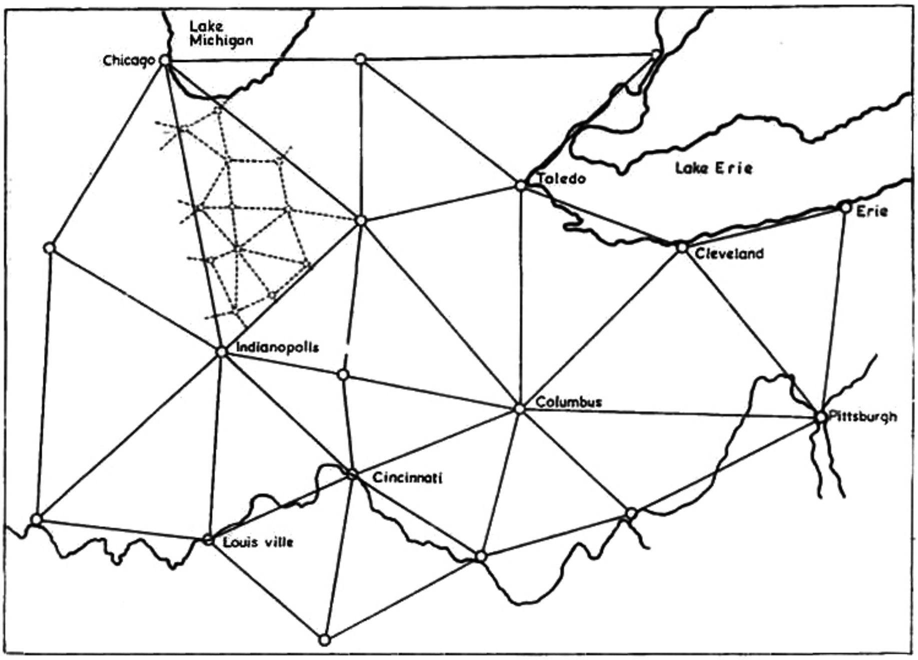


Fig. 3. Transport network arrangement modelled by Cyrus Kehr.

a national and regional scale, the rules were first defined by C. Kehr and A.C. Comey dealing with spatial planning [27]. Kehr designed the transport network for the United States. The network consists of triangles whose location and shape are determined by the distribution of various types of settlements which are the triangles' apexes. Typically, the triangles are equilateral. The author runs the lines of national transport from nodes taken into account in the national plan (centres, focal points) located on average 150–200 miles from each other. These are predominantly metropolitan cities. The remaining settlements scattered within the triangles are connected with a triangular network of local lines (Figure 3). A similar concept was put forward and developed by Comey (Figure 4). He suggests use of the hexagonal arrangement not only to affect the national and regional transport network but also to plan transport in settlements and industrial areas.

H. Haufe's comments on the hexagonal system are purely theoretical. As I have already mentioned, the author drew his conclusions by examining the characteristics of various geometric shapes against two postulates. The first one (the biggest area – circumference ratio) is most completely expressed in a circle where the area – circumference ratio amounts to r:2. The other postulate (complete covering of the earth's spherical surface) is fulfilled by the spherical triangle of the sphere octant. This triangle is formed by graphical projection of three mutually perpendicular axes of the sphere. However, neither the circle nor the triangle fulfil both postulates at the same time. (in the triangle, the area – circumference ratio amounts only to $\frac{r}{4}$). This property is enjoyed only by a regular hexagon owing to its triangular structure and the accompanying area – circumference ratio amounting to $\frac{r}{4}\sqrt{3}$. As the sources of traffic are equally distributed on the surface of the hexagon, their connections are mist economical in the case of six radial (longest) sections aligned at the same angle and forming three straight lines. Therefore, a hexagonal network transforms into a regular triangle network, most suitable with respect to use. The length of the sides of the hexagons which on a larger area form a hierarchy is not arbitrary but

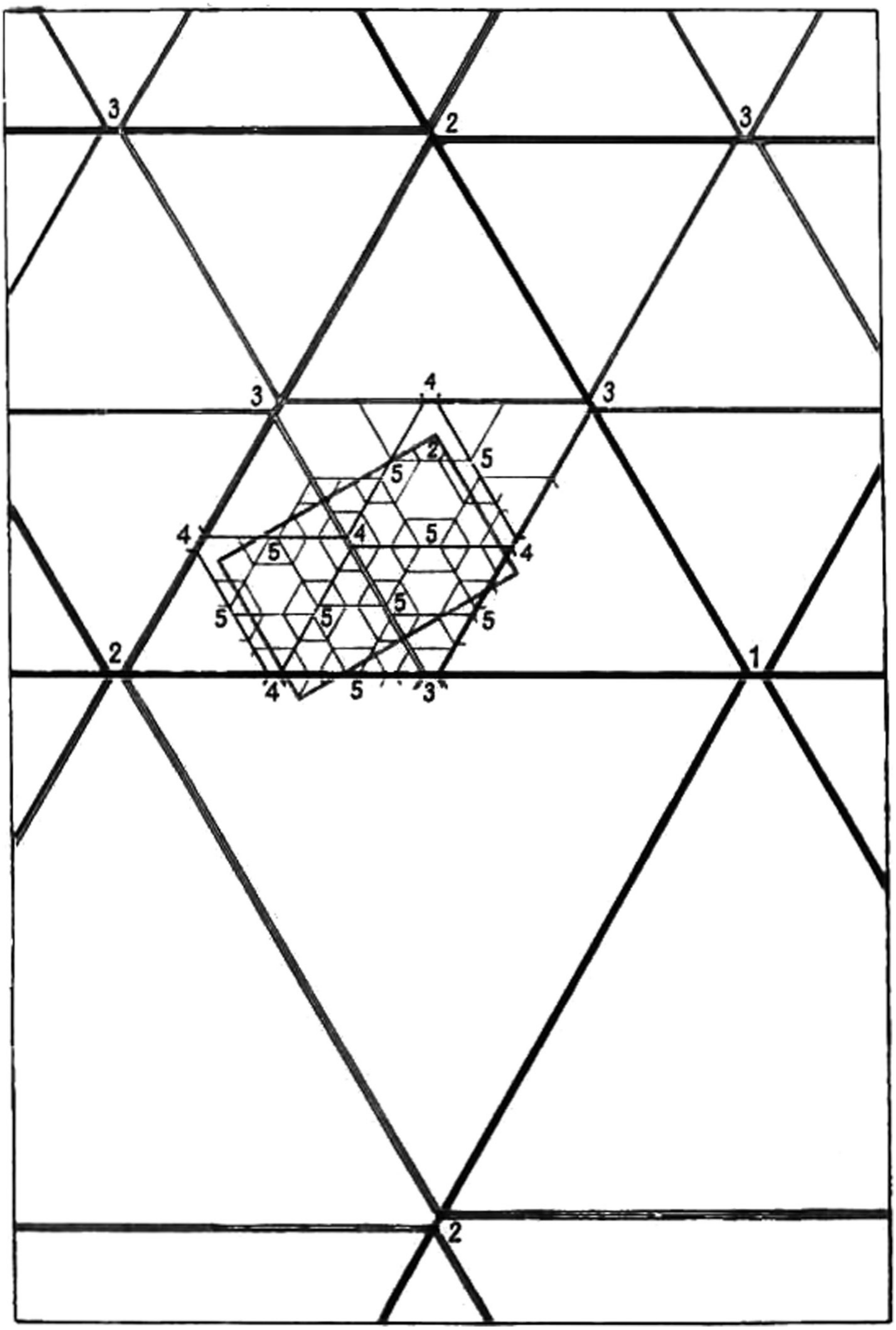


Fig. 4. A hexagonal transport network arrangement. I. A national arrangement. Modelled by A.C. Comey.

rather, determined by content expressed in kilometres (*an hour-long trip, a daily trip* etc.). The author practically disregarded the square which failed to fulfil his criteria (the area – circumference ratio = $\frac{r}{2\sqrt{2}}$).

Christaller presents two varieties of the transport network model: the first is derived from the market rule (supply) (Figure 5). In the case of the other one, he assumes that the rule of transport has an advantage

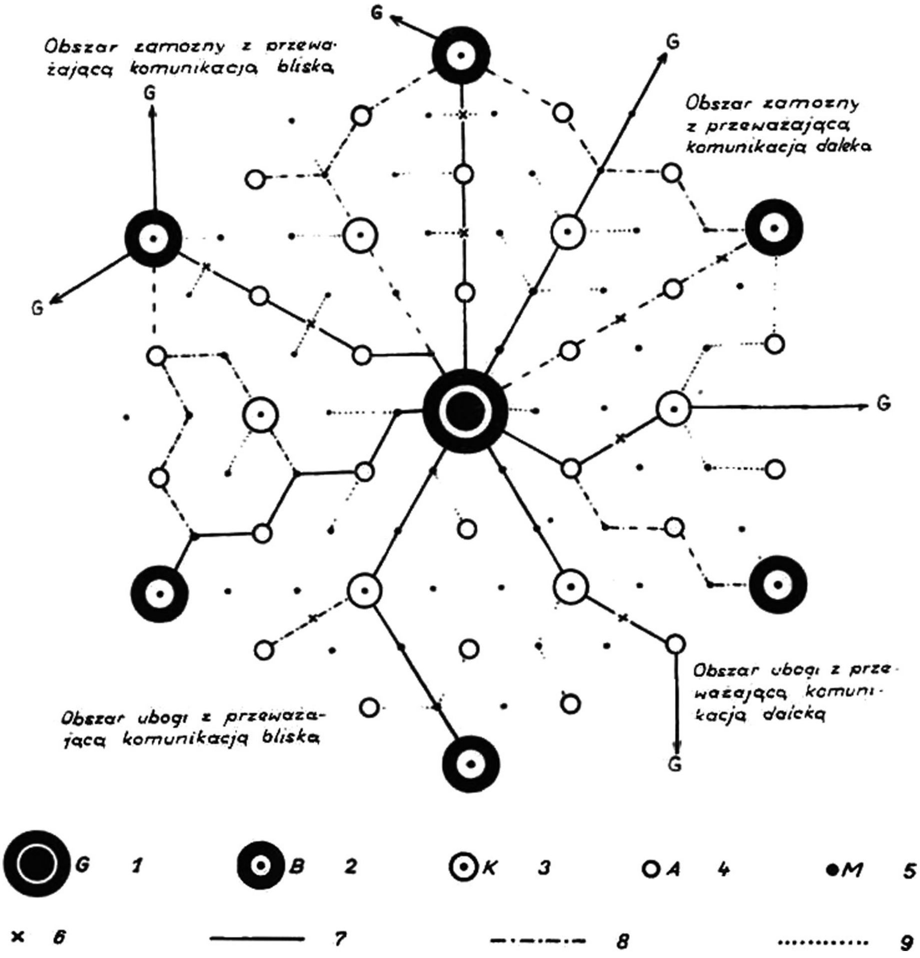


Fig. 5. Linie komunikacyjne w systemie ośrodków centralnych. Według W. Christallera

1–5 – typy ośrodków centralnych; 6 – miejscowości stacyjne; 7 – główne linie komunikacyjne; 8- drugorzędne linie komunikacyjne; 9 – łącznice.

over the market rule (Figure 6). In a system of central cities based on the market rule, the distribution of transport lines is not satisfactory. Straight lines, the best for traffic, are scarce and far away from many cities A,K,B[8]. As the author explains, one could give priority to the major lines and thus define straight lines between *G*-rank cities. In this scenario, however, within a single *G* system, each of them only crosses one *K* city and three *M* cities while *B*-type cities remain on the peripheries. Therefore, secondary lines need to be created for them which, if straight, connect with *G* on top of *B*, *2A*. Between every main line and secondary

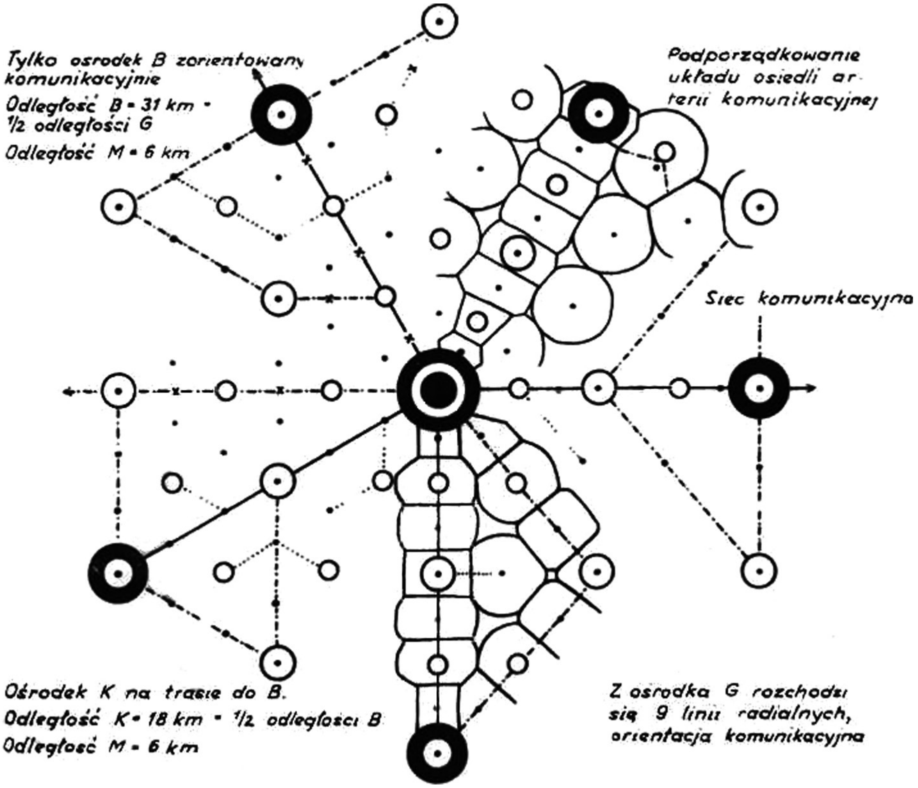


Fig. 6. A system of centres developed on the basis of W. Christaller's ideas of transport

---

[8] Christaller differentiates the following types of central cities: M – Marktort, A – Amtsort, K – Kreisstadt, B – Bezirksstandt, P – Provinzstadt, L – Landstadt.

line, *1A* and *3M* are located which can be connected with the main line by means of junctions or be left without a connection.

In the latter case, the central cities too far away from efficient transport (in Christaller's example over 3.5 km) may lose their central functions for the benefit of station settlements enjoying privileged transport location. Another trend to follow is the best connection between *G* and cities located at its back and identification as major lines connecting the most important cities. These lines are created mostly between *G* and *B;* when the route zigzags a little, they also connect *2A* and *3M*. City *K* gains a connection with *G* by means of a secondary line which further divides and connects the external *A* cities. The remaining *M* cities are served by junctions. More important changes occur in the system of central cities and transport lines where the transport rule dominates the spatial organisation of settlement. The changes are very clear on the main lines attracting more estates than the previous system without deforming the route. The line between *G*-*G* hosts 1*B*, 1*K*, 2*A* and 4*M* which are stage locations (according to the transport rule). In the remaining central cities of the single sector 1*K*, 1*A* and 2*M* connect with the other lines by means of secondary lines and 4*M* by means of junctions. The prevalence of the rule of transport is related to an increase in the number of centres which are indispensable for supplying an area with central goods of the suitable range. This phenomenon defies the market rule with its pursuit of maintaining centres on a minimum level. Both rules, theoretically well-grounded, in fact compete with each other resulting in three solutions: 1. the advantage of the market rule, 2. the advantage of the transport rule, 3. a compromise. The fragmentary nature of the transport network in Christaller's model is striking. Even if we assume that the author took into consideration only railways, the network is hardly general. This deficiency is even more striking in the face of the fact that the model relies on uniform handling of the area. Too abstract assumptions coupled with an ahistorical approach to the issue contributed to a limited application of Christaller's model, apparent especially with respect to strongly industrialised areas.

A different configuration is represented by the transport network in A. Lösch's model although the initial premises are very much like in Christaller's model. Following an analysis of the areas and network of various goods markets, Lösch made an attempt at ordering his observations. To this end, he arranged market networks in such a way that they have a common centre. Then, by rotating them long enough around that point, he came up with six sectors with numerous production sites and six sectors with few production sites. In this arrangement, according to Lösch, the sum of the distances between the productions sites (and con-

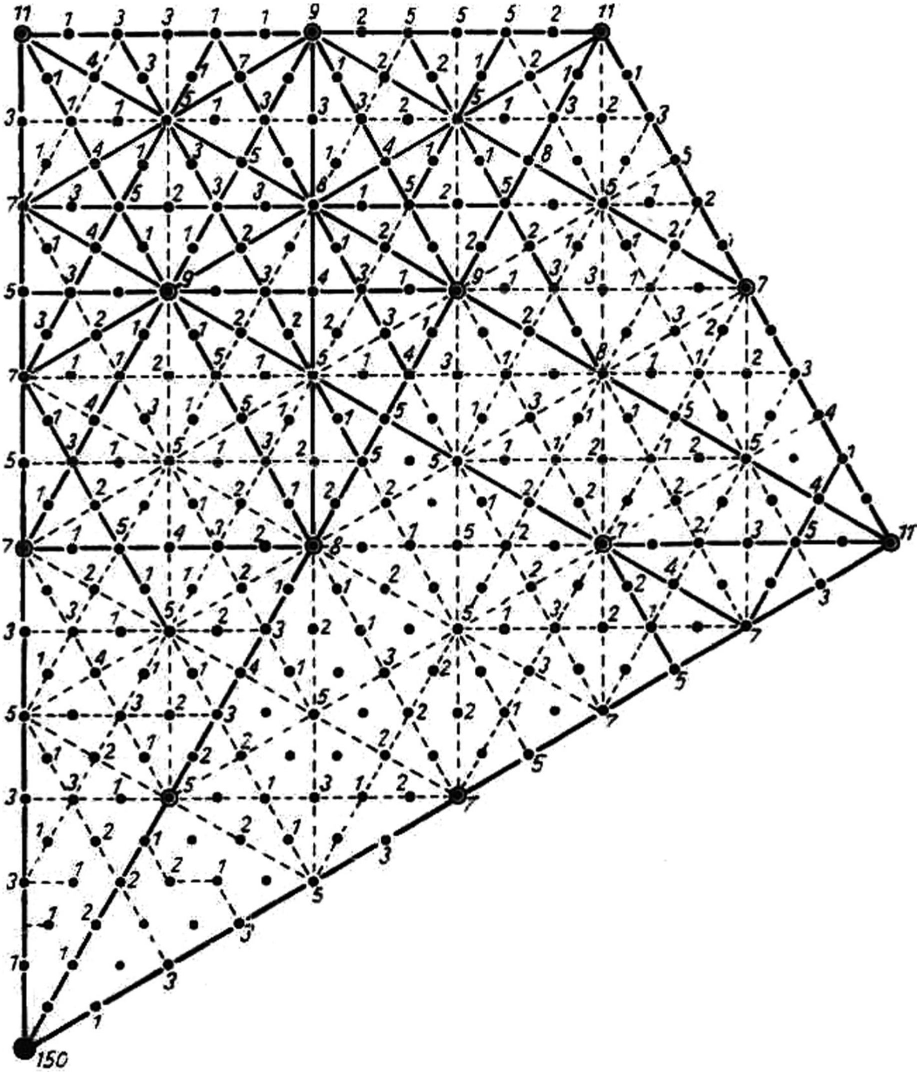


Fig. 7. Transport lines in an ideal economic landscape – one of the sectors). According to A. Lösch

sequently the sizes of loads and length of the transport lines) are kept to a minimum. In its general form, this theorem is certainly coherent. However, it seems that the transport network presented in Figure 7 does not fulfil the minimum condition. Other statements suggest that Lösch himself recognised the compliance with a honeycomb structure. In an ideal economic landscape, transport lines assume the shape of a cobweb. Twelve major radial lines come out of the centre, resulting from intersections of six straight lines. In other centres, only two or three lines intersect (again, the description does not comply with the drawing which suggests that in these centres, from two to six lines can intersect). Theoretically, the intersections in the immediate vicinity of the centre are meaningless and as such they were disregarded in the figure. The lines intersect at an angle of 30° and 45° or at angles equal to multiples thereof. As a result, the hexagonal layout is supplemented by right angles. Differences in traffic intensity are striking, both between the lines and sectors (with numerous and few cities). The scale is illustrated by the number of centres converging in the nodes. Along the thick lines, the number of centres is twice and more bigger while along the thin lines, the number of centres is approximately 1.5 times bigger than along the intermittent lines. The traffic is heaviest along the lines marking borders of sectors. In special market conditions, a square transport network may emerge although it is longer than the hexagonal network stretched in the same area.

W. Isard did not present his own model of a transport network. He only commented on it briefly while expressing his opinion on Lösch's concept. Isard was critical about Lösch because his geometric structure failed to express sufficiently the agglomeration trend in industry and the population, resulting from the benefits of mass production. Hence, Isard tried to modify the structure with this respect (and only this one) (Figure 8). According to Isard, the sizes and shapes of market areas change. As a result of a relatively bigger population concentration, around the main city and along the major transport lines the necessary and sufficient market areas are relatively smaller. They tend to grow as the distance to them grows i.e. where the population is more sparse and scattered. Maintaining the hexagonal forms of the market proved difficult.

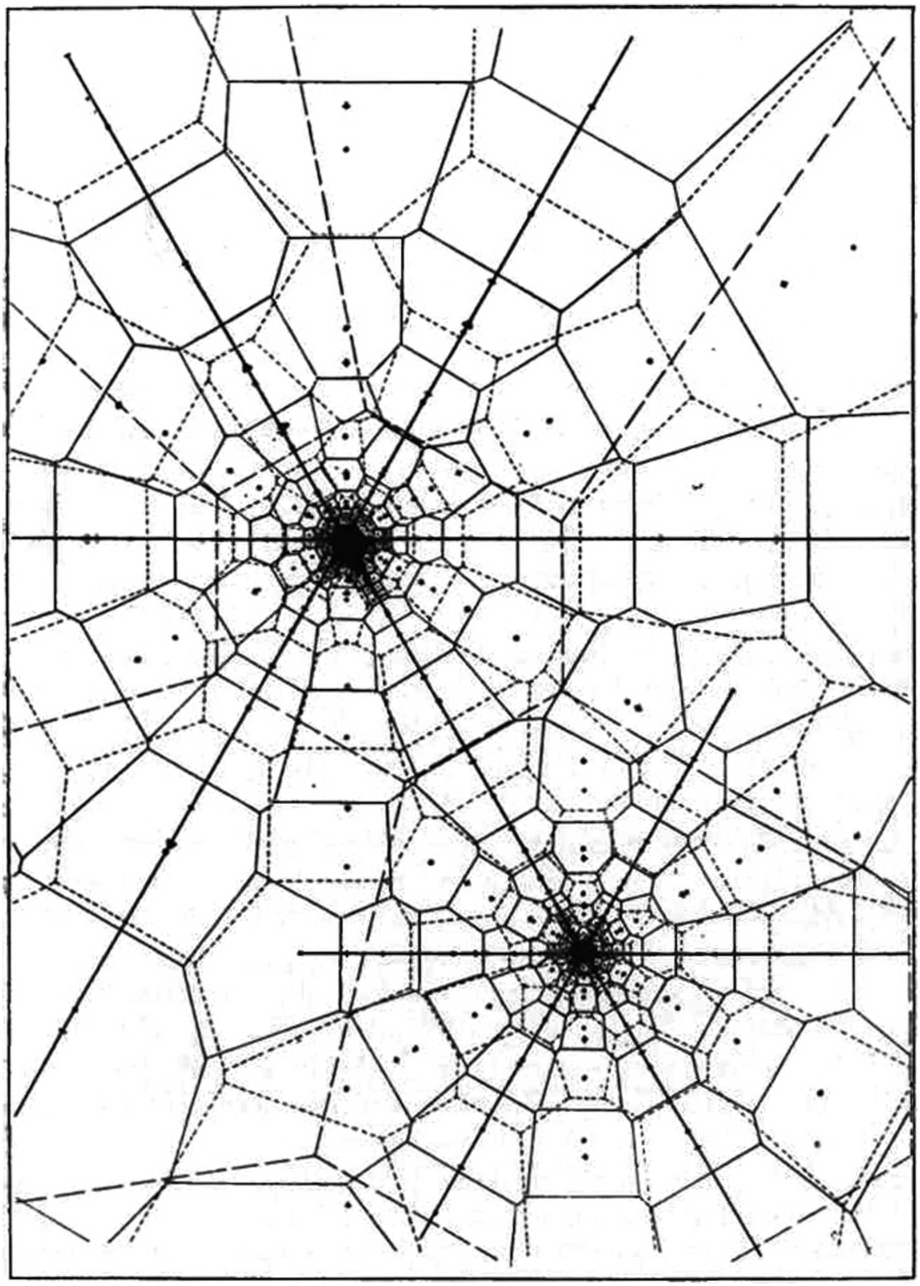


Fig. 8. Lösch’s modified system taking into account the population disytribution. According to W. Isard

According to Isard, these forms are appropriate in the conditions of perfect competition. However, when agglomeration trends intensify in the spatial system of economy, non-hexagonal forms are standard solutions. The transport network in the modified model is limited to six major lines; the secondary lines were disregarded just like the system of satellites. The location of the major lines is different than in Lösch's model. This is because the lines do not run along borders but through the centres of sectors with numerous and few cities. In this type of location, scale effects are stronger due to application of modern means of transport.

K. Dziewoński formulated the principles of a model layout of a transport network for Poland which, however, can be applied on a larger scale as a general model. The author corroborated his proposal with characteristics of typical natural and theoretical layouts. Each of them has its benefits and drawbacks as it is based on generally true but one-sided assumptions. Therefore, the model layout should be a combination of them, taking advantage of the benefits and eliminating the drawbacks. The way in which the specific layouts are coupled (*t*) depends on the development, technical condition and further prospects of a transport network. However, it may be only multi-layered, functional. On the other hand, spatial diversification is undesirable if in one region transport is affected by one layout and according to another layout in another region. According to Dziewoński, a model layout for Poland consists of: "a geographic arrangement when it comes to international transit..., a geopolitical system when it comes to the major network in the country..., a triangular system when it comes to the supplementary, separate or relay network, a concentric system when it comes to handling big cities, a hexagonal system when it comes to handling small towns, a rectangular system when it comes to the elementary management network, finally a functional system when it comes to coordination of the specific types of networks as a whole" [27].

Z. Wasiutyński's relevant concept is quite different as the point of gravity is shifted from the geometric patterns of a transport network, sometimes taken for granted, to methods of affecting the network's layout. The author started by postulating that the workload of traffic be reduced. Two mutually dependent factors stem from this postulate: 1.

Concentration of the population and production means in settlements, 2. Distribution of the settlements and their internal development as well as systems of city roads and roads running between settlements. New here is a combined overview of both factors by means of mathematical methods applied in technical sciences. In order to define clusters and distribution thereof by means of mathematical methods that would require minimum transport activity, both factors need to be formalised first i.e. their mathematical equivalents need to be identified and arranged according to the dependencies between the measures. The adopted equivalents included a system of cluster networks perceived as a system of points and a system of road networks interpreted as a system of straight sectors. The measures included lengths and angles, measures of transport mass and the related measures of transport density and relative transport density. The search of systems of road networks and transport goods responding to the postulate of the smallest burden is among extreme issues[9]. They can be solved by means of different methods depending on the form they assume. In the process of defining long-distance transport systems whose transport goods are viewed as concentrated in specific locations, geometry of a layout of points on a plane comes in. On the other hand, identification of urban transport systems (assuming that transport goods are distributed in a continuous way and roads are arbitrarily dense) is based on a mathematical analysis of continuous functions. By applying the right mathematical methods, the author introduced a theorem of existence of two extreme cluster systems: in one cluster, they forma square network and in the other, a triangular equilateral network. The roads connecting clusters form regular networks of lines. This theorem may be generalised in cases of clustered and evenly distributed goods, unevenly distributed goods, cases when the intensity of transport depends on reduction coefficients and superimposition of netweorks of clusters of different sizes.

The systems of lines and points deemed by humans optimum patterns for a transport network, are modelled by shapes created by nature. For example, the hexagonal system to some extent reflects a honeycomb (seen from above). The structure of a honeycomb is a work of art. If bees build cells other than hexagonal [35], e.g. round (like bumblebees) or octagonal

---

[9] These systems are referred to as extreme.

or pentagonal. There would be unused space between them and each cell would have to have entirely or partly its own walls (Figure 9, the upper row). It would be a waste of place and material. This can be avoided by building triangular, quadrangular or hexagonal cells with walls used doubly with no unused space left (Figure 9, lower row). Here, hexagonal cells have the smallest circumference while the capacity remains the same. To produce them, relatively least material is used therefore they are the most economical option[10]. There are numerous examples of hexagonal forms including colonies of green algae (Volvox) with flagella. The colonies consist of a large number of organisms (over a dozen thousand) connected with each other by threads of plasma to divide food, coordinate the movement of the flagella and the colony's fusion into a homogenous collective organism. Hexagonal crystals are a case in point from another area.

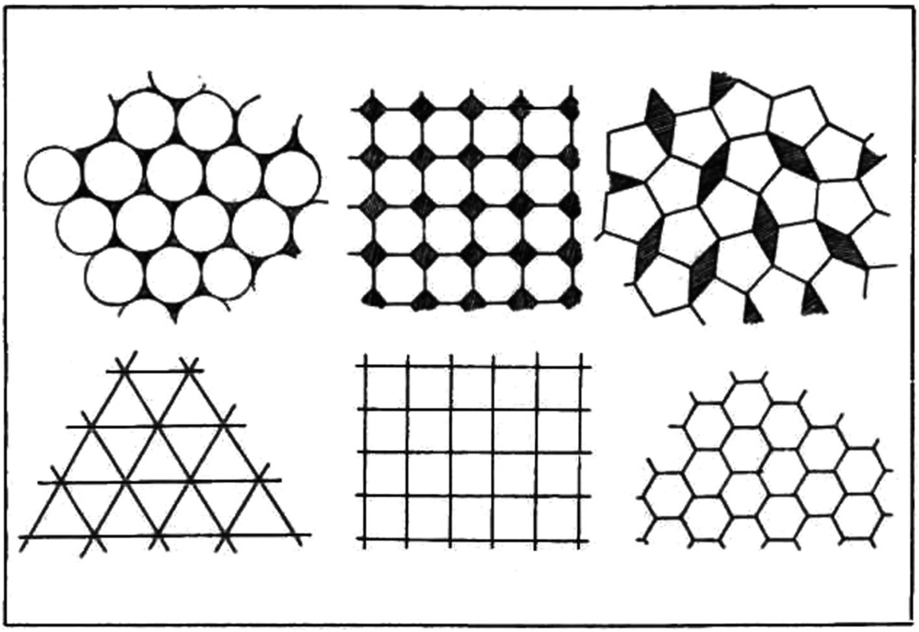


Fig. 9. Transport effectiveness of various geometric fugures

---

[10] Lepiej też odpowiadają walcowatym larwom pszczelim niż komórki czworo- lub trójkątne.

The similarity between a transport network and a cobweb, emphasized by Lösch and many other authors, is rather symbolic. This is because even in ideal conditions it is hard to imagine a transport network formed like a cobweb, without excluding the most sophisticated web spun by orb-weaver spiders (Argiopidae). The radial structure on which the spiral of the hunting web is stretched is reminiscent of a concentric transport system with excessively dense ring roads.

The functional transport system bears similarity to the human vessels system. Among the numerous organs, the pituitary gland and the kidney stand out for their extremely regular vascularity.

The analogies suggest that nature has created patterns also for complexes of networks of various types of transport. They can found among both physical and biological phenomena. Bearing in mind the development of various disciplines of science, adaptation of the laws of physics bodes better. The laws of reflection and refraction of light (Fermat's law) are undoubtedly suitable in fragmentary complex analyses. In a human body, an example of team work is the cooperation between the circulatory system and the lymphatic system. The division of labour between blood and lymph is more or less like that between the major and supplying means of transport. Blood carries nutrients to various parts of the body while lymph is a mediator between blood and tissues: it returns to cells the useful food load, at the same time taking used and harmful products. While biological organisms are bound to host more perfect patterns of complexes it is dubious if studies of them would allow to go beyond comparisons, attractive as they seem and superficial as they are.

Both in nature and theory, transport networks have a common feature, namely pursuit of minimising the amount of material and energy. Efforts are made to keep tangible assets and labour used to build a transport network at a minimum together with the transfer of goods at a certain level of transport needs. This condition (*caeteris paribus*) is fulfilled by a possibly shortest transport network (in the economic sense which does not always equal the physical sense). Therefore, in a spatial perspective, it is most important to minimise the ratio between the (economic) length of a transport network and the serviced area. This common pursuit results from the right to cross the space which can be referred to as the right of desire paths. In fact, in the realm of transport, it is the right of colloquial knowledge which does not necessitate detailed explanation.

Which transport network is most suitable for the requirements of the right? A mathematician would say that is a hexagonal system with its extension in the form of a triangular system where it offers the biggest number of applied benefits. However, triangles have a less favourable circumference : area ratio than hexagons. What is more, in this respect they are also inferior to rectangles. In a rectangular system, there are no diagonal directions which enforces peripheral traffic and increases the costs of network usage. In fact, a transport network between settlements tends to assume a triangular rather than rectangular system. The latter has been applied in newly developed areas (where a transparent network of connections is in demand) as well as small and lowland areas (where diagonal traffic is neither too intense or too costly).

The concept of a complex of transport networks

The meaning of a complex of transport networks tends to be unstable. This can be avoided by attributing the phrase's meaning by resorting to a design (regulation) definition. The understanding presented in this book is that a complex of transport networks is synonymous with a set of various types of transport roads which together form a whole because they operate for the same goal (transport for an area or a destination) accomplished by supplementing (complementing) or substituting them at the lowest total cost.

I shall refer to a regular, characteristic and correct system of the components of the complex as its structure while a system where the spatial dimension is of importance will be referred to as the spatial structure. The regularly repeated relations between the components of the spatial structure of the systems are referred to as the structure's law[11]. The spatial relations follow the law of coexistence; however, they are causally conditioned (they exist as spatio-temporal relations). Therefore, in fact spatial relations are governed by causal laws which encompass the law of coexistence.

---

[11] I use the term *strictly general theorem* or, shortly, *the theorem.*

Due to the spatial structure of complexes of transport networks, the constitutional features of complexes of transport networks include: 1. A hierarchy of roads, 2. the location of roads against each other, 3. linear exchanges (numbers and lengths of roads). These features combine in the complexes in various ways but this diversity reflects typicality. Complexes of transport networks vary with respect to the number of segments and the degree of structure. Complexes are two-segment (two-pronged), three-segment (three-pronged) and multi-segment (multi-pronged). Typically, the more segments the more complicated the relations between them (gradation of structure). There are complexes

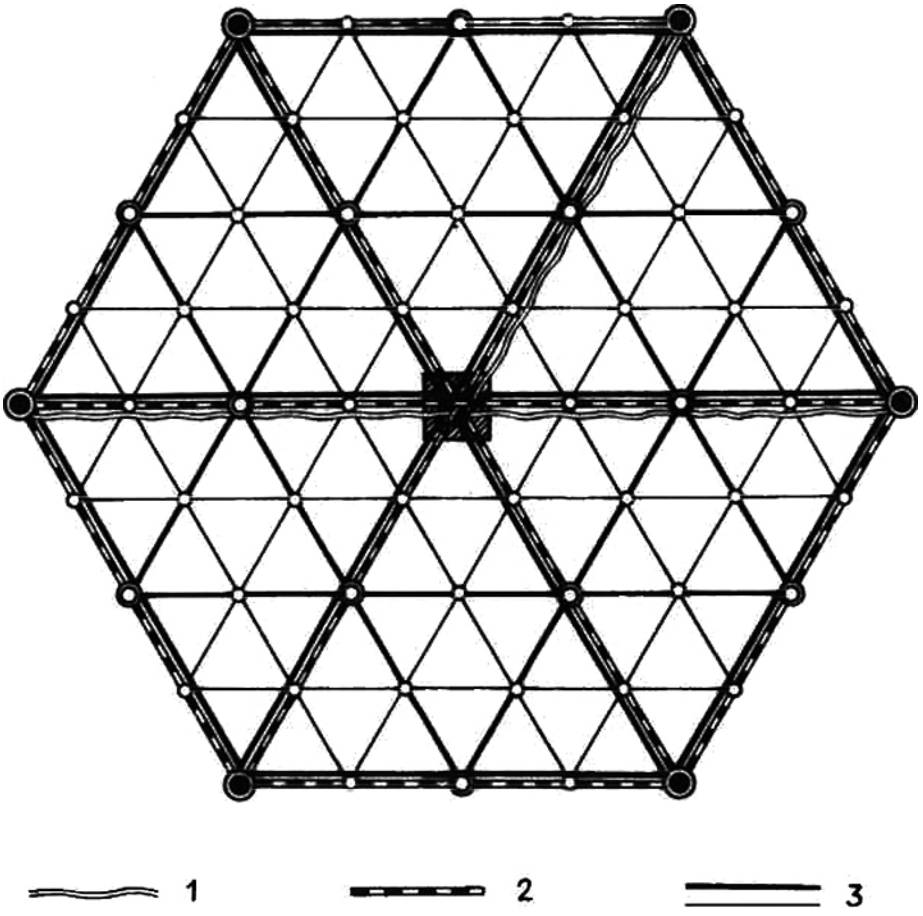


Fig. 10. A simplified model of a complex of transport networks
1 – rivers; 2 – railroads; 3 – roads.

– loose conglomerates and compact organic complexes. Even in poorly structured complexes there are lines and nods (complex axes) which, when immobilised, distract the entire complex's work or paralyse it. For this reason, the structure of a complex should make it possible (within certain limits) to change the functions of the specific segments in order to replace the immobilised segment if need be.

Following the law of the desire paths and the defined hierarchy of various types of transport, a simplified model of transport networks can be constructed (Figure 10). The complex's general layout is hexagonal. The functional diversity of the different types of transport is related to the hierarchical diversity of the lines of different levels[12].

A region's capital is a node of six railways, combining the capital with all the secondary cities. Parallel to them run the main roads tailored to heavy traffic. Between them is a network of supplementary roads which handle traffic to the railroads and major roads as well as direct local traffic. As it is, waterways do not form a complete system of connections. An assumption that one river with a right-bank tributary flows through a region probably is probably not inferior to any other assumptions. The major directions enjoy bigger investments; in order to handle heavier, functionally diverse traffic, multiple connections are formed, clusters of transport lines of different types.

The presented model is of course insufficient. Its major deficiencies include: 1. failure to take into consideration the different properties of various types of transport, 2. statistics, 3. assumptions of two-dimensional and homogenous space. These drawbacks indicate the directions of the future research aimed at building a developed and dynamic model of a complex of transport networks.

---

[12] The phrase "transport line" with reference to a complex model is presented not only in a technical but also, predominantly, a kinematic context.

III. Approximation

The approximation of solving the problem of complexes of transport networks will take place in two stages: 1. in the course of developing a theoretical complex and 2. In the course of agreeing on the typology of complexes. The first procedure to which this entire chapter is devoted, is based on an assumption that despite significant diversity, factors which affect complexes can be approached theoretically, including strictly general indicators. The result is a model of a theoretical complex. In identifying the typology of complexes, these factors will be presented less generally, classified into characteristic groups.

From all the factors I have selected the most influential ones. While they are not the only effective factors it is imperative to limit oneself to them; otherwise the work would be limited to solving single cases without a chance of the much awaited generalities.

1. The properties and functions of various types of transport. Allocation premises

The major factor in the emergence of complexes of transport networks is diversification of the functions of various types of transport. The diversification exerts forces which bind a complex internally. The functions stem from technical and economic properties and should be employed in shaping complexes[13].

---

[13] An induction analysis is limited to three types of transport: by rail, by road and by inland waterways. The most frequent processes are complementation and substitution; the roads and ways are permanently located and trigger off permanent changes in the space. Therefore, despite incomplete induction, the result may be considered (pars pro toto) sufficiently representative and meaningful.

The technical properties of all types of transport are defined by means of the properties of the specific components and the way in which they are coupled. Typically, three elements of transport are identified: distance, tractive force and vehicle. Road properties like the type of pavement, number of lanes (tracks) etc. are ultimately short-lived. They also include the number and distribution of the feeding points which also define provision of access to an area by roads. Roads for vehicle traffic have the biggest number of such points; they are even treated as made available to traffic along the entire length. This holds true for small-scale traffic on waterways. On the other hand, railroads are available only in strictly defined and relatively less densely distributed points; they shape economic space like a loose set of points rather than lines. Road capacity tends to be the opposite and for this reason increasing thereof typically necessitates reduction of the number of feeding points. For example, compared with regular types of roads, motorways have much fewer turnings which are avoided, just like in the case of railroads. The average capacity of the specific types of transport are characterised by the following numbers [67].

1. Car roads: at the speed of 40 km/h, with a ratio of passenger cars and lorries like 25 : 75, traffic irregularity of 75% – 1,440 vehicles per hour, the result is approximately 5,670,000 people and 1,620,000 tons a year.
2. Railroads: double-track – 67 pairs of trains per day i.e. approximately 24,000,000 tons a year; single-track – 23 pairs of trains per day i.e. approximately 8,300,000 tons a year.
3. Waterways: canals and canalised rivers – with single sluices, 20 minutes required for one train consisting of a tugboat and two barges (deadweight 1,000 tons each) to cross a sluice, traffic irregularity of 10% – 8,750,000 tons a year.

Roads tend to be more technically perfect when the resistance they put up to vehicles is smaller and as they require less tractive force to complete a specific transport. Then, with the same tractive force, it is possible to make use of bigger vehicles which allow to organize mass transport. The roads and ways of different types of transport necessitate transport of equal load of the following tractive forces (in arbitrary units): earth roads 250 units, paved roads 80 units, railroads 5–10 units, waterways 1 unit [39].

The size of vehicles measured by load capacity is also affected by other technical properties of the road: the width of the lane and the type of pavement (roads), acceptable axle load (railroads) as well as a cross and longitudinal sections (waterways). Technical progress makes it possible to build increasingly bigger vehicles. For example, in the United States 15t and heavier trucks are popular, capacity of rail wagons exceeds 60 tons while the average net load carried by trains frequently exceeds 2,000 tons (on the lines services by the Chesapeake and Ohio Railway Company, the maximum weight of a train amounts to 13,000 tons [110]). The same holds true for the deadweight of a typical barge transporting petroleum down the Mississippi [72]. As the capacity grows, the ratio of the vehicle's own weight and its payload improves. The resulting advantage may be considerable on condition of sufficient supply of loads. For this reason, large vehicles are launched for major destinations with concentrated transport. However, for scattered transports in a large number of points offering small loads, small vehicles are appropriate as they ensure better use of the loading capacity. In order to better adjust vehicles' load capacity to the supply of loads, road transport keeps diversifying its stock. In countries with large numbers of vehicles the share of cars in the average tonnage decreases while the share of small load vehicles (light commercial vehicles) and large load vehicles grows.

There are interdependencies between the specific elements of transport. Therefore, roads not only affect the required tractive force and size of vehicles but they develop under their influence. The car was a forerunner of modern changes in the technology of constructing and maintaining roads; the benefits offered by the car can only be fully enjoyed on good roads. Similarly, electrification of traction imposed new requirements on railroads. I mean upgrade of both the technical equipment and the courses of roads (the curve radiuses, longitudinal gradients, transverse slopes); the bigger the vehicles and tractive forces, the simpler the routes should be. Roads which are not upgraded together with vehicles are degraded to an extent when they are no longer used. The number of such cases is particularly large on small rivers and canals which used to be parts of a homogenous system of connections but later on, as a result

Table 1. Changes to the sizes of vehicles used in transport
(data provided in tons of load carrying capacity)

Time	Inland navigation	Road transport		Railroads		
	the Rhine	horse cart	car	Germany	UK	USA
1. Ancient times	50	0.75				
2. Middle Ages:						
1200–1500	50	0.75				
3. Modern period:						
1500–1800	150	1				
1840	250	3		4	4	10
1860	500	3		6	4	12
1880	700	3		10	6	20
1900	1000	3		15	8	36
1913	1800	3	2	15	10	40
1930	1350	3	5	16.5	12	42

Source: C. Pirath [85].

of the size of ships growing, the rivers and canals were no longer suitable for navigation and changed their status from connecting to dividing sections.

However, the degree of the composition of the ingredients varies from one type of transport to another. It is highest for the rail sector where the traffic is inherently related to the track and cannot be lateral; as a result, railroads are a rigid means of transport. The same can be said about inland navigation. The car has the great advantage of relative freedom of travelling which, combined with a more interconnected road network, makes it a flexible means of transport. These differences are reflected in traffic organization.

Among the economic properties of the various types of transport, costs are most expensive. A cost analysis will not suffice; the cost structure needs to be studied because the specific elements have different economic meaning and differently affect the spatial range of each type of transport. A structural analysis should include the relations between: 1. The costs of roads and the costs of transferring loads, 2. Fixed costs and variable costs[14] and 3. Costs of transport and costs of reload (the latter two are discussed in § 3).

The costs of roads and the costs of transferring loads change as the technical development of transport takes place in a discordant way. Roads, expressed in km, are becoming increasingly costly. At the same time, as roads are more advanced, their capacity grows together with the number of transported loads; the growth is more than proportionate. As a result, while the costs of roads grow absolutely, they relatively fall. On the other hand, the share of the costs of relocation constantly grows, accompanied by a drop in the individual total costs). In particular, the costs of vehicles grow faster than the costs of roads. This phenomenon is a part of the evolution of every type of transport, more so when old types

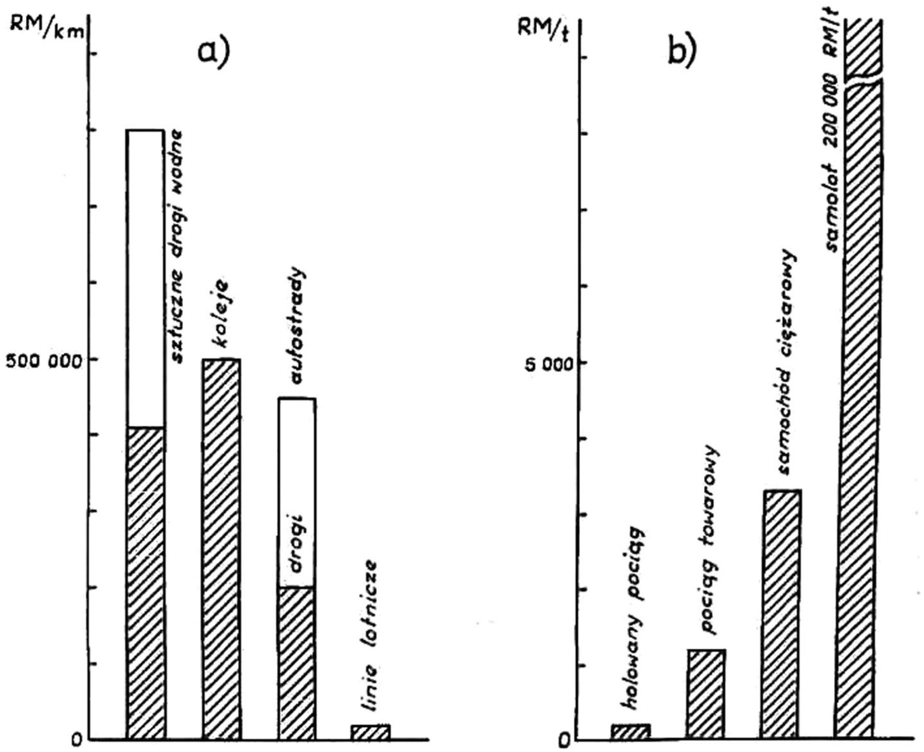


Fig. 11. Investement expenditures a) per 1 km of a two-way road and b) pet 1 ton of a truck's capacity

---

[14] Podobne znaczenie ekonomiczne ma podział kosztów na stacyjne i liniowe.

of transport are replaced by new ones. They are illustrated on Figure 11. Multiplied expenditures on vehicles are incurred to equip them with new features, benefits. Due to further generalisations, attention should be drawn to the gradual loosening up of the relation between a vehicle and a road; this extends the range of transport to unbeaten tracks, makes it possible to adjust vehicles of one type of transport to the traffic on secondary roads thus it is conducive to the development of direct transport (without reloading).

However, roads have such a potential that transport outrivals a majority of areas of social economy with respect to the share in the fixed assets of the value of devices which, *mutatis mutandis*, may be compared with roads. An incorrectly developed network of roads involves therefore massive losses which need to be topped with losses incurred in the process of transport (when vehicles are driven along roads whose shape deviates from the optimum one) to fully realise the significance of the regular theory of a transport network. Many authors have handled the issue yet neither of them has formulated a theory that could be applied also for complexes of transport networks. In fact, there is no transport network in general but a number of various networks which complement and replace each other.

The spatial range of the various types of transport is probably most affected by the relation between fixed costs (or costs independent of traffic) and variable costs (dependent on traffic)[15]. Fixed costs, calculated per unit of transport work (tonne-kilometre) fall down as the distance of transport grows. Therefore, the bigger the relative share of the costs, the longer the radius of rational range of a specific type of transport. There are major differences in this respect between inland navigation and railroads (bigger share) on the one hand and car transport (smaller share) on the other hand. Compared with railroads, inland navigation has an equal or bigger share of fixed costs on artificial roads and a small-

---

[15] These concepts are regularly used in transport literature yet use thereof requires an instant comment. Namely, fixed costs are actually fixed only to a limited extent while variable costs respond differently to an increase in transport; they can be proportionate, less than proportionate or more than proportionate.

er share on natural roads (contemporary navigation is typically accompanied by expensive investments in waterways).

Share of fixed costs in the general costs of transport (%) [75]

A.	Railways	61
B.	Midland navigation	
	natural waterways	44
	man-made waterways	61
C.	Car transport	
	trucks	32
	ACT trucks	50
	buses	40
	ACT buses	46
	passenger cars	54

Many attempts have been made to define the radius of each type of transport's rational range. Different results have been provided, reflecting the diversity of the factors at play in the specific countries and regions. The cost of the tonne-kilometre (comparable) is regarded the main contributing factor. The degree to which vehicles' capacity is used is the correcting factor. It varies from one type of transport to another and tends to be less important in rail transport and more significant in car transport (according to Pirath, [86] it amounts to 50% and 75%, respectively). This is because cat transport has at its disposal smaller and more agile vehicles, bigger opportunities of return loads and enjoy a greater freedom of decision-making (own, non-profit transport prevails where regular traffic is not required). On the other hand, railways are encumbered with a number of public functions which do not ensure sufficient use of the capacity. What is more, in a typical spatial arrangement of rail transport large numbers of wagons used to carry coal, the major load, are used only in one direction without return loads. As a result, car transport can extend its range of operation.

Below I present the results of some works on a rational range of various types of transport. According to Pirath [85], rail transport of small parts loads are less expensive than car transport only when the distance exceeds 150 km. In the case of full-load transport when a better use is made of the wagons' capacity, railways are cheaper when the distance exceeds 55 km. When rail transport is topped with the cost of delivery

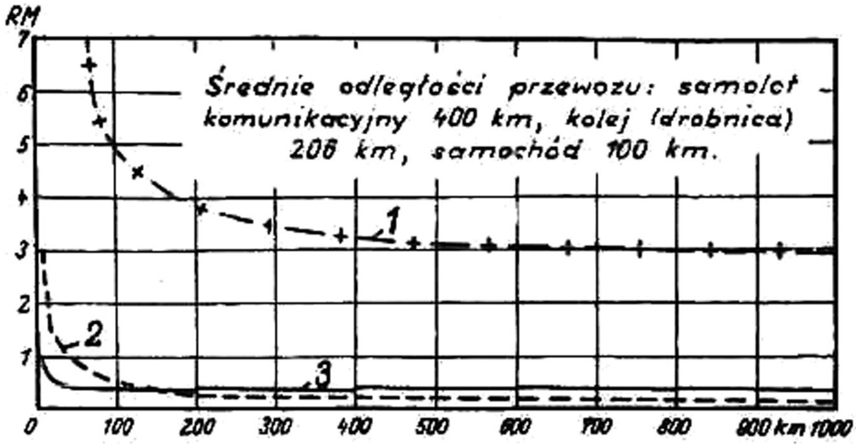


Fig. 12. Comparison of transport of small loads by different means of transport, according to C. Pirath

1 – scheduled flights – aeroplanes filled 50%; 2 – railways, small loads, filled 20%; 3–5-ton trucks, filled 50%.

and pick-up and cars transport goods “door to door”, the range of car transport moves from 55 km to 65 km (Figure 12).

Natural waterways are less expensive than railways starting from 120 km in no-reloading transport and from 200 km in transport with reloading. On man-made waterways, for transport with reloading it is from 400 km and for man-made waterways – from 580. This is how Morgenthaler and Wollert [74] identified the points of intersection of the railway cost curve and the car transport cost curve which define distances to which it is more economical to use cars:

railways	cats	km
5 tonnes	5 tonnes	140
15 tonnes	14.5 tonnes	170
20 tonnes	20 tonnes	200

They were of an opinion that in a foreseeable future, the range of car transport stops at approximately 200 km. This is determined by the structure of car transport costs. The barrier could be crossed only if the structure of car transport costs bears similarity with the structure of rail transport costs which is not very likely.

Research into transport in the Mississippi Valley has shown that in full truckload transport, the truck is the most effective means of transport over a distance of up to 35 miles; railways over a distance of 35–380 miles and inland navigation over a distance of more than 380 miles [42].

In the USSR, as E.D. Chanukow provides [17], direct truck transport along good roads and with relatively small loads, are more profitable yhan train or mixed transports (train-truck):

- over a distance of up to 30 km if the loading and unloading sites have industrial spurs;
- over a distance of up to 50 km if the industrial spur is only on the loading site or the unloading site, over a distance of 80–100 km in the case of lack of industrial spurs.

N.A. Lukyanov [69] calculated that the costs of transporting kerosene products via water with a single load (guaranteed depth of 150 cm, the coefficient of extending the watercourse of 1.5, barge deadweight tonnage 2,000 tonnes), equal the costs of direct transport by primary single track railways (relevant altitude 9‰, 2-axle and 4-axle cisterns) to a distance of 590 km. When barges with deadweight tonnage of 6,000 tonnes are used, watercourse transport becomes more advantageous after covering barely 255 km; for barges with deadweight tonnage of 12,000 tonnes – from 440 km.

Polish scientists have calculated that car transport is less expensive than rail transport on a distance of up to 30 km if the latter is combined with delivery to a train station and up to 50 km where delivery to the station and pick-up are required [73].

What functions do the specific types of transport perform with respect to their technical and economic properties?

Since car transport is relatively more branched but the roads are less capacious, the vehicles have smaller capacity, the traffic elastic, fixed costs represent a small share, car transport should ensure delivery and pick-up of loads to and from the train station, delivery of food from the suburban zones to cities, delivery of goods from warehouses to retail outlets, to make available areas without railways with scattered loads (especially arable land, forests, mountains), should participate in seasonal transport (autumn peaks etc.), take over transport on unprofitable local railways, provide short-distance and LCL transport and even on roads parallel to railways as well as handle passenger traffic: suburban, local, far from railways, and tourist traffic.

We should differentiate between auxiliary transport (delivery and pick-up) and independent (direct) transport. Their relations and developmental trends are evidence of a deviation from a simplified complex of transport networks and should become a part of the premises of an anisotropic syndrome (chapter IV). For want of the related statistics, I have viewed interchangeably the relation between short-distance transports and short- and long-distance transports[16].

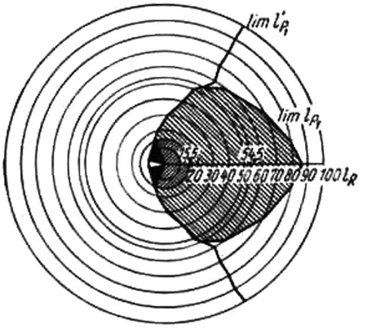
Short-distance transport typically takes place in a two-direction system as [87]:

1. Radial transport aimed at direct carriage from where the goods are dispatched to their destination along the shortest possible route.
2. Circular transport taking place both on radial and peripheral lines. It typically consists of three sections: radial (to), circular and radial (back). Two forms of this transport have developed: a. destination circular transport (handles only three estates at the interface of sections) and b. collective and separate circular transport (handling a larger number of estates).

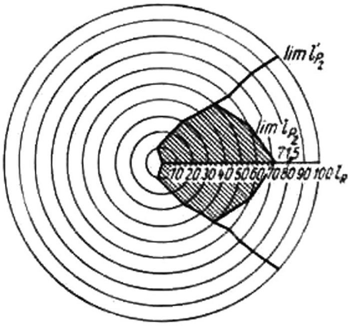
To date, car transport prefers radial directions with considerable traffic as they are likely to provide return loads. However, due to dispersion of transport loads, circular (especially collective and separate) transport works better in agricultural areas. The limits of destination and collective circular transport, juxtaposed with the cost criterion and time, are presented in Figures 13–16. They show the areas where circular transport is more profitable on either side of the main (radial) road. The shapes of the areas result from the fact that in the vicinity of the initial location and then from the maximum point, as the radial distance

---

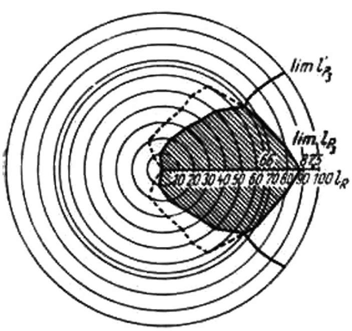
[16] This exchange is justified, making the subsequent conclusions even more justified. This is because short-distance transport includes also direct transport. This can be easily checked if we consider that in the quoted statistics, the following distances were defined as short: in France up to 49 km, in the UK up to 40 miles, in West Germany up to 50 km i.e. distances largely exceeding the average distances of car transport. Therefore, the exchange decreases the share of direct transport while the conclusions are based on increase thereof.



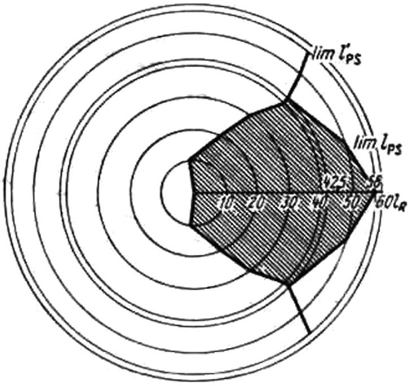
13



14



15



16

13. Economic limits of circular transport in a 5-ton truck, the costs and time included (case 1)

14. Economic limits of circular transport in a 5-ton truck, the costs and time included (case 2)

15. Economic limits of circular transport in a 5-ton truck, the costs and time included (case 3)

16. Economic limits of circular collective transport in a 5-ton truck, the costs and time included (4 reloads)

1 – extended peripheral transport; 2 – regular peripheral transport; 3 – limited peripheral transport; 4 – radial transport. Case 1 – trip to destination loaded, peripheral trip empty, return trip loaded; case 2 – radial and peripheral trips loaded; case 3 – trip to destination loaded, peripheral trip loaded, return trip empty

grows direct radial transport becomes more profitable. As a result, the circular distance from the main road diminishes.

While car transport is relatively most effective on short distances, in specific relations and groups of loads it has an advantage (albeit diminishing) on the other types of transport on medium and even long distances. This opens up an economic opportunity to extend car transport routes which is actually happening. In Western Europe medium- and long-distance transport (calculated by tonne-kilometres) represent or exceed 50% of all car transport routes (Table 2). Since car transport is still at the expansion stage, the proportion between short- vs. medium- and long-distance transport may continue to change for the benefit of the latter, especially in smaller countries.

Owing to the considerable capacity of railways and the wagons' load capacity, due to a big share of fixed costs, small traffic and dispatch elasticity, railways are intended for mass transport, especially when combined with industrial spurs (when reloading, delivery and pickup can be avoided), long-distance transport, handling long-distance express passenger trains and mass commuter traffic. In most countries, industrial spurs process a majority of loads carried by trains. For example, in Poland it is 60% [73], in the USSR 80% (loading) and 65% (unload-

Table 2

Factors	Medium- and long-distance car transport			Heavy vehicle transport[1]
	UK 1952[2]	France 1954	West Germany 1954	Italy 1954
Volume in billion tonne-km	13	13	13	18
Percentage of all car transport	42	59	52	65
Percentage of rail transport	35	32	30	136
Average transport distance in km	134	127	232	137
	Average distance of rail transport			
All distances provided in km	126	246	189	263

[1]All transport (including short-distance) by trucks of capacity exceeding 5 tonnes

[2]Excl. Northern Ireland

Source: [31].

ing) [98], in West Germany 68% [91]. Despite eliminating poorly loaded spurs[17] and the new technology of multimodal car and rail transport, the share of spurs handling rail load will grow rather than diminish as a result of the railways limiting themselves to transporting mass loads. This change to the scope of work carried out by railways will also affect the arrangement of lines and stations as well as the structure of rolling stock. The existing rail networks will transform into less dense networks consisting of the main lines adjusted to intense traffic. Resistance to axle load will grow more or less [96] to 30 tonnes, the train's weight to 6,000 tonnes while a locomotive's power to 6,000 KM.

Local, lightly loaded railways will be gradually eliminated. For decades, they played an important role as an intermediary between major railways and small, dispersed centres, carrying economic centrifugal forces to rural areas and, consequently, positively affected the economy's spatial structure (contributed to decentralisation thereof). However, local railways tend to show regular financial deficits. This leads to a conclusion that now its functions could be taken over by car transport. The change in roles should take place following survey of each route separately. Too hasty decisions to eliminate transport routes made with reference to general ratios may undermine the economic structure of typically less developed regions. In the regions examined by Pirath (northern Württemberg, northern Bavaria and Lower Saxony), replacing local railways by car transport would have some repercussions for passenger transport and other consequences to transport of goods. In the first case, the costs of freight to be borne by the transport users would hardly change; in the other case, they would grow but differently than in short- and long-distance transport. In short-distnace transport (up to 100 km), the cost of local rail transport, when transport and reloading at a station are required, more or less equals the cost of substitute truck (direct) transport. On the other hand, if customers of local railways make use of an industrial spur, switching to truck transport involves bigger costs unless 15-ton trucks are used. In long-distance transport (over 100 km),

---

[17] The approximate criterion of an industrial spur's profitability is the goods mass loaded or unloaded within a year. It is estimated that the mass should amount to 40,000 tons a year or 5 loaded wagons per day (sometimes 3 wagons per day are considered the bare minimum).

the extra costs are higher even if the best scenario following dissolution of local railway connections is adopted i.e. transport by trucks to the main train station and from there by train to the destination (Figure 17). The costs would be highest if the entire transport were by truck (due to the moderate transport needs the use of the capacity would not exceed 50%) not to mention the fact that in less developed areas, the required number of trucks might not be available[18]. The financial situation of local railways could improve and their operations could be extended by means of technical modernisation. Between Annecy and Albertville [84], following employment of small vehicles and simplification of traffic organisation, the deficit was nearly eliminated; the connection between Albertville and Doussard (to which further research was limited) even proved profitable (the traffic load amounted to barely 150 tons per day).

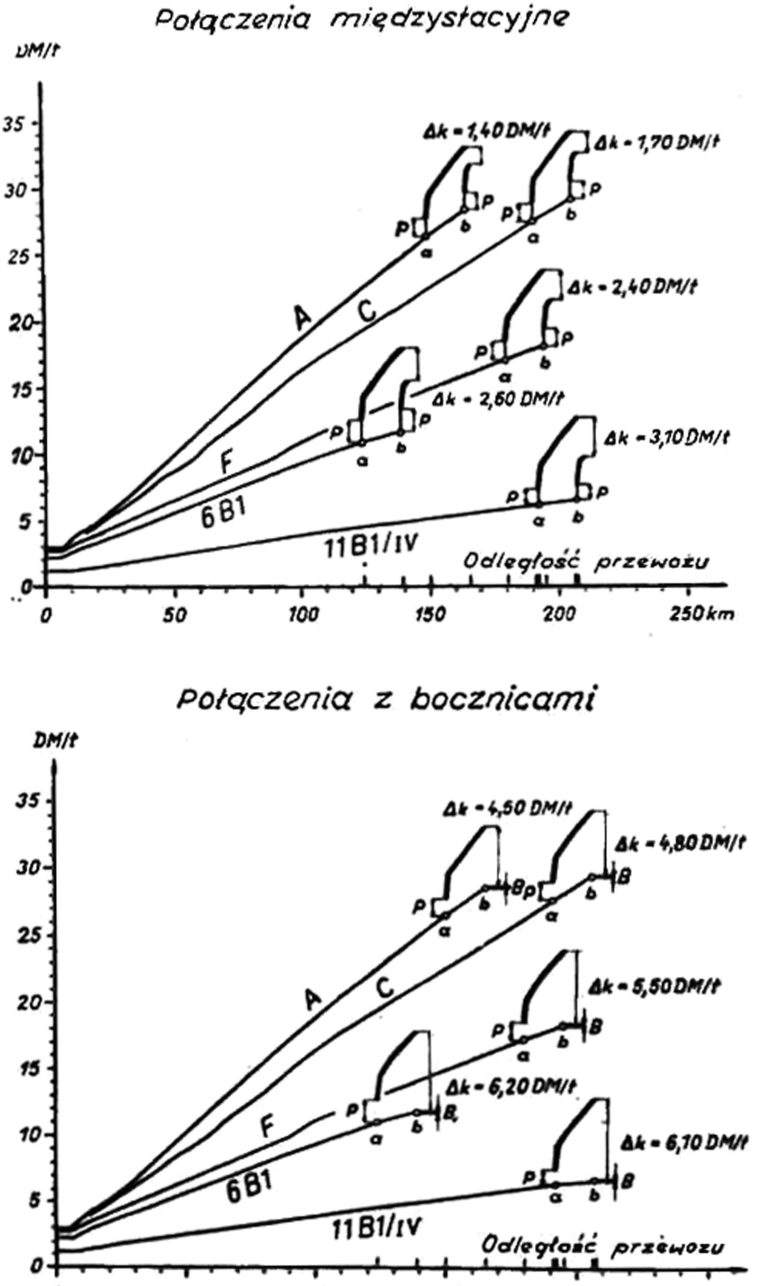
Despite the technical differences, inland navigation resembles railways, at lewast with one economic respect. Namely, inland navigation has the same or a higher share of fixed costs. This property affects the partial similarity in the scope of the function. Therefore, just like railways, inland navigation is used for mass and long-distance transport but to a greater extent becasue as transport mass and distance grow, unit costs keep going down. Of equal importance is the fact that inland navigation handles loads that can be cheaply and quickly reloaded (grains, sand and gravel, oil, coal). The low cost of transit itself is fully demonstrated[19]. However, due to the small speed of traffic and the typically longer waterways[20], the process of transporting bulk products requires relatively more time. As a result, inland navigation is not suitable for carrying perishable or valuable products. The advantage of inland navigation is even greater when the starting point and the destination are

---

[18] Pirath's calculations were distorted by failure to coordinate railway and truck tariffs.

[19] Inland navigation is a cheap form of transport – for example shipping 1 ton of coal from Buffalo to Duluth located 2,000 km away does not cost more than carrying all that coal from a street to a cellar [39].

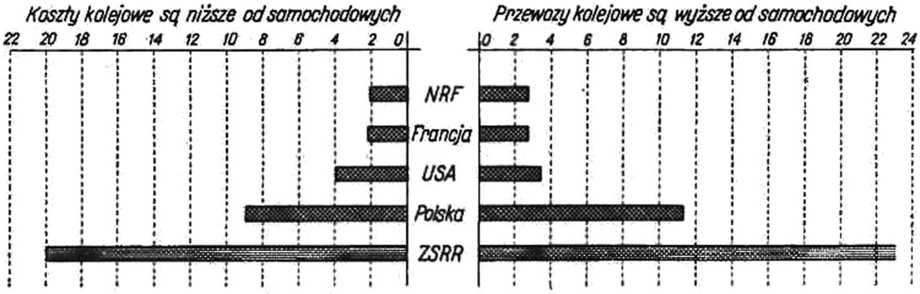
[20] This pertains to natural ways. If the topographic and technical conditions allow, canals are almost straight. Track design of canals is different than track design of roads: a faraway place is not connected by changing the route of the main canal but by means of a fork in the form of a side canal.



17. The difference between transport before and after closing down side railways in long-distance shipping according to C. Pirath.
P – cost of rloading; B – industrial spur fee.

located near waterways. The advantage grows for transport in relations with sea harbours provided that reloading goods from barges on sea shios is less expensive than from train wagons on sea ships. Inland waterways represent an indirect type of transport between land and sea shipment. They take the advantages of sea transport inland and, at the same time, they provide a natural connection between the land and the sea. In poorly developed countries occupying large territories and suffering from lack of railway connections (especially when the territory is not easily available like tropical forests), rivers remain the major means of transport and give way to edonomic development.

The technical and economic properties affect not only the quality but also the numer of functions of the specific types of transport. The quantitative scope of the function is affected mainly by prime costs. The lower the prime costs of a specific type of transport, the bigger its share in general shipping and the other way round. Of course this relations is not simple; it is further complicated by preventive or prohibitive tariffs imposed on a specific type of transport, route or distance. Sometimes it is difficult and too expensive to expand transport with lower unit costs, e.g. extension of railway nodes in agglomerations and conurbations. In this situation, it pays to take over shipping that blocks nodes, even if the unit costs of substitute (truck) transport are higher. This issue is explained by the theory of comparative costs. Despite all these complications, the relation is clear enough (Figure 18)[21] to lead to a hypothesis



18. The reversed proportions of transport volumes and costs of railway and truck shipment

---

[21] The drawing developed on the basiss of publications [1], [17], [23], [38], [72], [87], [105].

that could be referred to as a theorem about reversed proportions of transport volumes and costs when using different types of transport. This rule applies more as the shipped mass grows and the availability of the compared types of transport increases and as their development is more level (due to the sporadic occurence of waterways, the relations is reflected only in the railway: truck ratio).

2. Elementary complexes

In an analysis, a scientist needs to identify the simplest, regularly repeated structural relations in a chaotic structure perceived in a superficial observation of complexes of transport networks. These relations include multimodal transport and the relations between it and direct transport. They are demonstrations of the complementary and substitutive nature of shipping. In fixed network infrastructure, they are expressed by means of the relations between main roads (cheaper transport) and side roads (more expensive shipping). One main road and one side road, when connected, create a lowest-level elementary complex. In contemporary complexes, relations between railways and roads prevail and pose the buggest numer of problems. Interestingly, the relations between main and side roads have not been explained yet sufficiently while they remain useful i verifying the theoretical model of a complex.

Having established the elementary relations, an academic needs to define the development of the budding structure of complexes. There is a numer of factors which determine and thus explain the structure, the most significanyt including 1. The reation between the costs of the various types of transport, 2. A series of complexes and the duration of the directions, 3. czynnik anisotropy. Now, a dependence will be presented between the structure of the complexes and the costs of the various types of transport.

According to the rule of management, multi-modal transport is only justified if the total cost equals or is smaller than the cost of direct transport. However, it is false to claim that in multimodal transport, the drive for minimising the cost results in forwarding the load as long as it is possible by a less expensive road and as short as it is possible along an expensive road. This is because the more expensive roads (byways) would be located at a right angle against the less expensive (main) roads which would not always fulfil the minimum condition. Therefore, how should these roads be located against each other?

The solution is based on the so-called law of refraction in transport which is a paraphrase of refraction in physics. When you adopt the law of refraction as formulated by Fermat as the starting point (Fermat's principle), a formal analogy is complete. Fermat's principle says that the path taken by a ray is always extreme i.e. minimum, maximum or stationary. In an non-homogenous environment, the path taken by a ray equals the sum of all the elementary paths taken by a ray and is expressed by means of the integral

$$l = \int_A^B nds,$$

where n denotes a fixed refraction coefficient, ds – sections of the path taken by a ray small enough to consider the refraction coefficient fixed, A, B – points of the path taken by a ray. The extreme condition of the path taken by a ray is for the integral variation to equal zero:

$$\partial \int_A^B nds = 0.$$

Let us assume that G is a border between two environments, a thicker and a thinner one (Figure 19). The the path taken by ray AMB is

$$l = n_1 AM + n_2 MB,$$

where n_1 and n_2 are the refraction coefficients of either environment. By converting this equation we arrive at

$$l = n_1 \sqrt{a^2 + x^2} + n_2 \sqrt{a^2 + (c - x)^2} .$$

Let's calculate the derivative of this function. A path reaches its minimum when

$$\frac{dl}{dx} = n_1 \frac{x}{\sqrt{a^2 + x^2}} - n_2 \frac{c - x}{\sqrt{a^2 + (c - x)^2}} = 0.$$

However, as the drawing suggests,

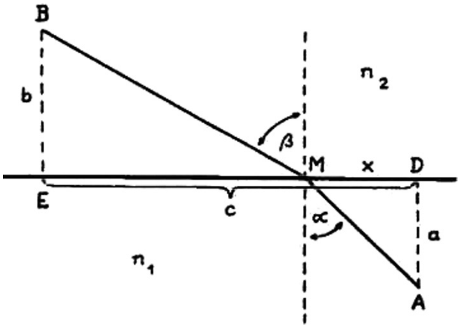
$$\frac{x}{\sqrt{a^2 + x^2}} = \sin\alpha \text{ and } \frac{c - x}{\sqrt{a^2 + (c - x)^2}} = \sin\beta.$$

Hence

$$n_1 \sin\alpha = n_2 \sin\beta \quad \text{or} \quad \frac{\sin\alpha}{\sin\beta} = \frac{n_1}{n_2}.$$

The transition from the law of refraction to the law of refraction in transport is based on changing the premises. Two optical environments are replaced by two different yet internally homogenous transport areas separated by a straight line. The refraction coefficients are replaced by the total unit costs of both types of transport. As a result, multimodal transport is the cheapest if it refracts in such a way that the sines of the refraction angles and the refraction of paths are inversely proportional to the total unit costs of both types of transport.

However, in this expression, the law only explains simplified transport relations, namely the relations between the points located on homogenous planes, equally accessible everywhere. This expression needs to be further converted to highlight the single-dimensional, linear elements which correspond with the nature of transport. To this end, we need to introduce the concept of the critical angle. This angle cannot ex-



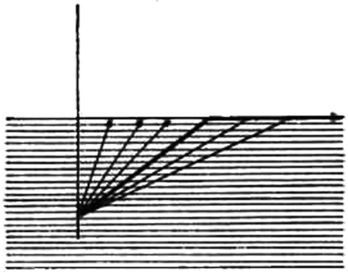


Fig. 19. The path of a refracted ray
Fig. 20. The critical angle

ceed 90° while the refraction angle equals 90°. In this case, the refracted ray runs in parallel with the border of the environments while $\sin\beta = 1$. Hence, on the basis of the law of refraction, we arrive at a critical point formula $\sin\alpha = \frac{n_2}{n_1}$.

This formula allows to determine the location of byways (more expensive) against the main roads (less expensive). This is because the transport-related interpretation of the dependence is as follows: the direct byways from a specific point in an area of more expensive transport are the most effective connection with the main road if their angles with the perpendicular to the main road are not bigger than the critical angle. Multi-modal transport to the remaining points on the main road is the most effective i.e. forst down a byway inclined to the perpendicular to the main road at the critical angle, followed by transport by the main road to the destination (Figure 20).

Let me now define the location of vehicular traffic roads against railways by assuming the transport relations prevailing in areas considered typical. In Western Europe, 1 tkm in transport taking place on large distances by large volume vehicles (including the costs of the roads) amounts to approximately [31] 2–3 US cents. In railroad transport, the equivalent cost of 1 substitute tkm revolves around 1.5 cents and to around 1.8 cents including the capital interest rate. Since car transport conforms to the same rules as any new technology we can expect that in the future, the related costs will go down more quickly than in the case of railroad transport. I assume – cum grano salis – that for a foreseeable future, the ratio between the railroad costs and car transport costs will amoun to 1:1⅓ (1.5 and 2.0 cents).

While this may sound like a paradox, in the United States the span of railroad and car transport costs is broader. This stems from the relatively higher costs of car transport as well as the low cost of railroad transport. The US railways have largely depreciated old fixed assets; at present, less capital – intensive directions of technical progress are preferred (e.g. by avoiding electrification). What is more, they railroads are burdened with heavy traffic. On average, the cost of 1 tkm amounts to 9–10 millów while in car transport, for the same transport categories, it is 3.5–4 cents [72]. In the face of a remarkable development of car transport, the relative drop in the related costs in the future cannot be large. Let's assume that the ratio between railroad and car transport costs, decreased to 1:3, turns out to be rather sustained. In poorly developed countries – the former colonies (Africa, south Asia, South America) car transport plays an important role due to the insufficient capital required to build railways and the small scale of transports. Car transport has already enjoyed good financial performance. For example, in India 1 tkm in car transport costs 37–53 paisas and in rail transport 21–39 paisas [30]. But for the high taxes imposed on car transport, the cost ratio would probably be the same as in Western Europe. In these vast countries, the rail-car complexes of transport are still rare, giving way to the more popular and typical traditional land and water complexes and ones where the main roads are more or less modern (railways, motorways) while the roads and the means of transport are extremely promitive (paths, carriers, pack animals). This technical contrast resulted in a span of unit costs amounting to 1:10 and more (even if cheap labour is taken into advantage).

Let me mark the ration between the cost of rail transport and car transport as while the critical angle of the railway with the perpendicular to the railway as . Assuming the unit costs at the indicated height and substituting the critical angle formula, the following results for the three mentioned cases are obtained:

$$\lambda_1 = \frac{1}{1\frac{1}{3}} \quad \sin\alpha_1 \approx 50^\circ$$

$$\lambda_2 = \frac{1}{3} \quad \sin\alpha_2 \approx 20^\circ$$

$$\lambda_3 = \frac{1}{10} \quad \sin\alpha_3 \approx 6^\circ.$$

Figure 21 is a geometric illustration of the value of function sin . It shows how – when the unit cost ratio changes, the gradient of car roads changes against railways This gradient is measured by means of a slope of car transport roads (modelled by the slope of a straight line), $m = \mathrm{tg}\varphi$.

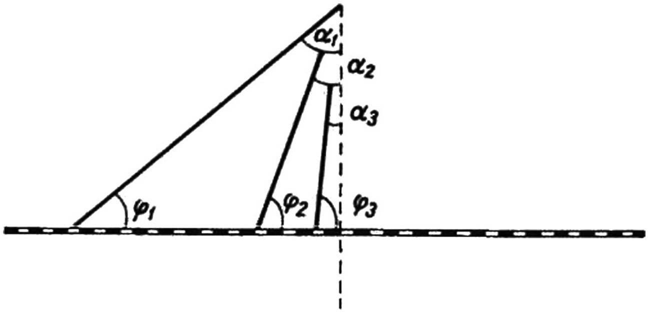


Fig. 21. An elementary complex. The location of a car road against a railway

Jego wartość wynosi: tg$\varphi_1 \approx 40°$, tg$\varphi_2 \approx 70°$, tg$\varphi_3 \approx 84°$. Therefore, the bigger the unit cost ratio the smaller the gradient of car roads against railways. Wen the span of costs is very big, car roads aim at a position perpendicular to the railroads. This is logical because, other things equal, the more expenisve transport is the shorter the access roads should be i.e. the closer to the perpendicular position.

3. Evolution of complexes of transport networks

Observation of the evolution of complexes of transport networks leads tostrictly general statements which prove helpful in explaining their spatial structure[22].

As for the position of various transport roads against each other, fixed relations can be observed, further referred to as the parallelisation theorem. Namely, in their position against the roads of the old types of transport, roads built for new types of transport initially aim at a perpendicular position and, later on, through transition stages, to a parallel position. Therefore, in relations to rivers and canals, railways were originally intended to connect or supplement them; for some time, railways developed next to tchem to finally replace them. The same process is taking place in the relations between roads and railways. Railways have changed the functions of roads: they have taken over transport to the main destinations, changing the role of roads to access lines. Zmieniły się w związku

---

[22] The observations will only take this course.

z tym trasy inwestycji drogowych[23]; zmniejszyło się (względnie) inwestowanie kierunków głównych, a wzmogło inwestowanie dróg w kierunkach prostopadłych i promienistych do stacji kolejowych. However, when the development of the motomotive system was significant enough, the investment routes turned to directions paralel to railways.

These cyclical changes reflect the changes to the ratio between the prime costs of the competing types of transport. Initially, the typical means of transport involve relatively higher costs; what is more, properly developed networks are not readily-available. Therefore, they initially perform auxiliary access functions; roads tend to be positioned perpendicularly because then their length and, subsequently, the cost of construction and use, is minimum. Gradually, the technology develops, prime costs go down and investment funds are accumulated for fixed assets. New types of transport take over the independent functions and, spreading spatially, they prevail in paralel directions. The first to succumb are destinations insufficiently serviced by the old types of transport or burdened with various transports requiring transport with different properties (when they form clusters of transport roads).

Parallelization increases due to changes in the ratio between the transport costs and the reloading costs. In the past hundred years, the transport costs of all types of transport (and therefore the tariffs) have

Table 3. Reduction of tariffs in various types of transport in 1840–1950 (*Pf*/tkm)

Type of transport	1840	1913	1950
Inland navigation (the Rhein)	4.80	0.41	0.47[1]
Rail	14	3.58	6.8[2]
Roads[3]	35	26	11[2]

[1] 1930.

[2] At current prices.

[3] 1840 horse transport, 1913 and 1950 – car transport.

Source: E. Sax [94], C. Pirath [851, [87].

---

[23] In the evolution of rails in Europe in modern times, enhancement of the technical equipment is more characteristic than the development of destinations. This is why parallelisation is more distinct in the change of road investment routes than in an arrangement of routes which in its natural form emerged earlier. The same holds true for the laws of plateauing to be formulated in chapter IV.

been seriously reduced (Table 3). For example, rail tariffs are ate least ten times lower than carriege fees charged by horse transport on the onset of the development of rail transport (at comparable prices); special tariffs may be twenty or more times lower. As for reloading, technological change has brought about a significant increase in devices' loading capacity and a shortened loading time, yet failing to decrease the costs significantly. As a result, the reloading costs became a growing burden to transporting goods. At present, in rail transport, in many countries the loading costs increase the transport costs by 50–100%[24].

This distribution of costs triggers off an increase in direct transport (reloading-free). This trend is reinforced by a drop in the value of goods, inevitable in the course of reloading. For example, the value of carbon goes down by 8–10% [85]. Therefore, direct transport can eliminate costs higher than the sum of the costs borne in multimodal transport (transfer to and back, reloading, transport). This extends the potential range of direct transport. At the same time, technological change offers the means because it differentiates the properties of means of transport which makes it possible for tchem to adjust to the varying transport sizes and requirements for different groups of loads and to use direct transport for the most suitable goods and in the most suitable area. This adjustmjent process leads to specialisation with all its benefits. All the three means of transport, especially rail and road transport, are involved in direct transport, to a large extent between same i.e. large or smaller cities and towns. In other words, this is transport in the same directions therefore requiring parallel routes.

Direct transport between cities located along the main road and off that road, rather distant in the longitudinal direction, aim at an acute-angled arrangement. This trend is also reflected in the road arrangement. As a result, traffic can move along the hypotenuse instead of the legs. In the case of considerable longitudinal distance and a small transversal distance, the angle between a side road and a main road may be very acute. This is a case of quasi parallelization.

---

[24] In the US, by approx. 190% (1952–1955). This result has been obtained by assuming that the cost of transporting 1 ton-mile amounts to 9–10 millów, the average distance of transporting 1 ton amounts to 430 miles while the cost of reloading 1 ton is $7.8. The starting numbers as in [72], [32].

Despite the high costs of reloading, there are vast areas of multimodal transport because the range of costs offered by a less expensive and a more expensive means of transport tends to be so broad that the benefits of the leass expensive means compensate for at least the costs of reloading[25]. What is more, multimodal transport is a technical necessity in the face of the fact that the roads within the network of less expensive transport are less condensed and less forked than the road networks in more expensive transport. It is interesting how the reloading costs affect the arrangement of access roads (more expensive transport) used in multimodal transport, against the main road (less expensive transport). Namely, these costs lead to a deviation of the access roads to a perpendicular position against the main road. When they are added, a part of intermodal transport ceases to be profitable. The spatial range of this transport narrows down in a way where transport from peripheries is eliminated. What remains is transport from destinations located better in relations with the main road station. Notably, the location's attractiveness (and the transport's profitability) grow as their distance from the perpendicular line to the station diminishes.

Therefore, the elementary complexes devised in the previous paragraph solely from the ratio between the costs of vatious types of transport, require two explanations: introduction of paralel roads and the deviation of side roads toward a perpendicular position against the main road. Both trends and the resulting transformations of the elementary complexes are presented on Figure 22 (the top part like in Figure 21). In order to identify the deviation of side roads resulting from the reloading

---

[25] In the US [72], when it comes to mass loads, the least expensive is transport on the Great Lakes by ships with 20,000 tonne payload and by river barges. In 1955, the cost of 1 ton-mile for ships amounted to 0.6125 milla (full cost) and 0.4375 milla (current expenses) when the loading capacity is used in both directions; 1.225 milla and 0.875 milla when the loading capacity is used only in one direction. In the case of barges, when they are not fully loaded on their way back it is 2–2.5 milla (full cost). The equivalent rail costs (prices from 1952–1955) amounted to 4.1 milla (wagon with 60,000 pounds payload) and 3.9 milla (wagons with 70,000 pounds payload) when the wagons are loaded in both directions. Therefore, even when the assumptions are bedst for the rail and worst for inland navigation, the latter remains much cheaper.

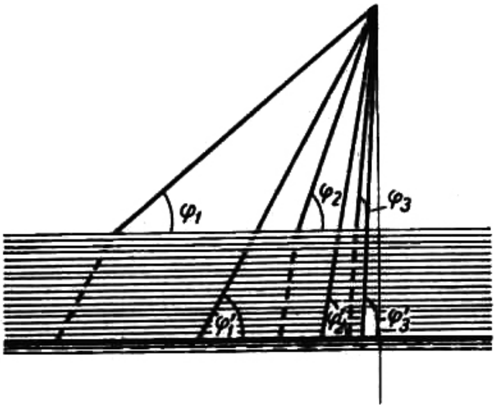


Fig. 22. An elementary complex. Deviation of a road resulting from the costs of re-loading

costs, we can make use of the familiar refraction formula. This is because the reloading costs create some sort of a new environement divided by a new boundary. The density of this environment grows at the same rate as the transport costs grow because of reloading. Therefore, the directions of side roads need to be more refracted, along the new boudary. The angle of the refraction depends on the angle of incidence of the roads and the ratio between the costs in a dense environment and the reloading costs and in a non-condensated environment (λ'). The corrected angle of roads against rails are defined by φ'_1, φ'_2, φ'_3. Data on the costs of re-loading and rail transport suggest that $\lambda_1' = \frac{1{,}75}{1}$ (the re-loading costs increase the rail transport costs by 50–100%, on average by 75%), $\lambda_2' = \frac{1{,}95}{1}$ (the cost of re-loading double the cost of rail transport (by 190%, by 95% of single transport), $\lambda_3' = \frac{1{,}75}{1}$ (like in the first case). The refraction angles amount to: sin $\alpha'_1 \approx 29°$, sin $\alpha'_2 \approx 10°$, sin $\alpha'_3 \approx 3°$. Stąd tg $\varphi'_1 \approx 61°$, $\varphi'_2 \approx 80°$, $\varphi'_3 \approx 87°$.

The other trend may change in the future under the influence of the new technology of mixed rail and road transport. Right now, four new forms of this type of transport are developing: 1. Transport in CONtrailers (piggyback, Huckepack) i.e. in trailers or semi-trailers, placed with ther load on flat cars, 2. Tranpsort in huge containers with craneless reloading, 3. Transport by means of transporter wagons (Strassenroller) on which entire wagons could be rolled along streets, 4. Transport in two-way wagons suitable for road and rail transport alike.

By eliminating the costly reloading where various types of transport meet, the new technology greatly improves the effectiveness of mixed transport[26] and therefore expands its spatial range. This may increase traffic and stimulate investments on steep access roads[27].

The relations between the importance of main roads and distribution and access roads (representing different types of transport), measured by the traffic and technical equipment also develop in a cyclical way. Notably, there is an analogy with the cycle of industry density and dispersal as identified by R. Maunier [70] and with technological changes as generalised by L. Mumford [76]. By the era of rail and steamship transport, the diversity of importance in the complexes of land and waterways is small; transports are dispersed while the technical equpment of the main and minor roads hardly varies, typically failing to meet the standards developed by Appius Claudius with respect to roads. Progress was made only in the realm of construction of vehicles. Because of the technical properties, the price and tariff structres, rail and steamship transport contribute to concentration of transport to the main destinations, typically heavily invested in. The diversity of importance is increased considerably because, simultaneously, secondary destinations develop poorly. Traffic on small rivers nearloy disappears. Navigation on rivers and canals paralel to railroads has declined[28]. In the entire network of

---

[26] Owing to CONtrailers, the cost of reloading 1 tonne where the goods are dispatched and collected, which in the US amounts on average to $7.8, goes down to less than $ 0.6 [32].

[27] Another result may be a partial dissolution of industrial spurs in cities, first in compact districts.

[28] Transport along the Erie Canal, between Buffalo and New York, dropped from 4.6 mill tonnes in 1880 to 2.385 mill in 1906; in 1880 this type of transport represented only 3% [94] of the transport provided by the competitive New York Central System and Erie Railroad Company. The main-Danube canal was obstructed by rail transport soon after it was built.

waterways, only single main routes available to large barhes ans ships, have fought off competition from railroads. The changes in the road networks are quite the opposite with main roads losing their importance[29]. However, access roads are also occassionally disregarded as a result of extending industrial spurs and local railroads. In modern transport complexes, concentrating trends have been slowing down to give way to the budding dispersing trends. While railways and the main rivers continue to develop their transport potential, especially in new countries, they are inferior to roads (access and distribution roads as well as long-distance) with respect to the rate of growth.

Adjusting roads to transport needs takes place by gradually providing technical equipment of the same type of roads and introduction of various types of transport. Therefore, the number of roads leading in different directions is growing in complexes, accompanied by a growing number of routes in single directions. However, this process may continue only up to a certain limit; once it is crossed, the number of routes may decline for the benefit of quality. Modern transport technology allows to cease to build extra rail tracks (third, fourth) on heavy traffic lines or even reduction of the existing rail tracks. This is possible owing to signalling control equipment (beside electrification, dieselisation etc.) which significantly increases the traffic capacity of rail transport. These devices are particularly effective in areas at a disadvantage from the point of view of the topography and the climate (deserts, swamps) where it is difficult and expensive to carry out construction works or main-

---

[29] Back around 1850, several hundred carts would cross every week Jordanów on their way to Orava [13], transporting salt from Wieliczka and Bochnia to Hungary. The goods transported on the way back included ores from Hungary and Spiš for the ironworks in Maków, Sucha, Żywiec etc. Later, these transports stopped, not as a result of the direct competition from rail transport but rather following the decline of industry in Galicia (Eastern Europe) which could flourish owing to the isolation from the industry in the West. The construction of the Alpine rails disproved the existence of numerous mining settlements whose inhabitants transported people and goods across the Alps. These settlements were depopulated while the remaining inhabitants had to switch to other jobs (wood industry, tourist services). The formerly busy transport routes almost slipped to oblivion.

tain personel (e.g. the Union Pacific Railroad in south Nevada). In road transport, with respect to traffic capacity the motorway means nmore than two regular roads and, while it is extremely expensive (on average, \$300,000–\$500,000 per 1 km), it may be profitable in the busiest routes between cities.

The desired spatial structure of historical complexes emerged in the proces of selecting roads (directions). Well-located roads stood the test of time and maintained their directions, despite the changes to the forms of transport. Poorly-located roads, built with transcient needs in mind etc., would become degraded or eliminated. Ancient Romans possessed a considerable skills of defining good locations for roads and cities[30]. Their cities were established in such good locations that, despite numerous damages and fires, they would revive in their original locations. The same holds true for the destinations of the Roman roads which are very clearly reflected in the contemporary European transport arrangement. However, earlier history also offers numerous mentions of roads which survived for millennia. For example, Hesiod mentioned the road between Kyllini (Elis) and Arcadia, visited regularly by merchants; Herodotus mentioned the excellent road from Sardis to Susa, whose length amounted to 13,500 stadions (2.500 km); Strabo mentioned the road between Pataliputra (now Patna) and the Indus River, 20,000 stadions long; Valmiki was familiar with the roads from Ayodhya deep into Punjab and the Allahabad area. The designs of the modern motorways and the Panamerican Railway are references to the main road in the Inca Empire, leading from Quito to north Chile. The old mule route, along which the Transandine Railway runs through the mountains of Bolivia, was also frequented by the Inca.

Selected directions create networks; their arrangement is more regular as the side effects wear off. The form assumed by the system largely depends on its origin and conditions of evolution. In the conditions of an organic evolution, an endogenous system tends to be triangular; it has achieved its most regular form in Europe. An exogenous system, formed in an arbitrary way and most characteristc of areas colonised

---

[30] Ancient Romans also had special road maps; the most famous was *Tabula Peutingeriana* from the 4th century BC.

by Europeans, tends to be rectangular (most consistently in North America), or amorphic.

The directions of transport networks have been favourably shaped by the gradual loosening up of the relations between a vehicle and a road. The relations with a car is more casual than with rail transport; the relations with an airplane is more casual than with a car (especially a helicopter or a VTOL aircraft), a relations with a hydrofoil (hovercraft) is more casual than with a river ship (a hydrofoil can be raised out of the water to be loaded or unloaded and it can be used on shallow as well as steep rivers). A means of transport less limited or not limited by rigoid rail tracks can move in directions which ensure minimum expensitures on transporting mass loads. The communist planned economy is destined for exploring this possibility in extreme complexes of transport networks.

When asked if, in the face of the volatility of the sources of movement, a search for lasting extreme complexes is sensible, I would answer that it is. It has been proven that it is possible to define shapes of transport networks whose parameters change only slightly, affected by the changes in the distribution of the sources of movement. What is more, in some conditions these shapes can be regular. P. Friedrich [104] reached an interesting conclusion in the course of analysing this issue. He identified a function that can be an indicator of how lasting is the network system's adjustment to transport needs. The function is as follows: $x = f(A, N, g,...)$ where x indicates the parameter of the network's shape, A –investment costs, N – traction wire cost, g – numer of trips. The durability grows as parameter x responds weakly to the changes to the measures A, N, g.

The durability of the system i salso typical of transport network systems; this is even more surprising in the face of the fact that while a system is operational, various forms of transport disappear and transform. This phenomenon can also be explained by means of the ratio between the costs of various cooperating forms of transport or, to be more exact, by the permanence of this ratio. While I formerly mentioned the cyclical changes to the cost ratio as the reason for the changes in the location of roads against each other (toward parallelisation). However, if we do not view this ratio in the various stages of the same cycle but rather, in the same stages of subsequent cycles, namely in stages of stabilised growth, the durability is quite distinct (despite the changing absolute costs). This is is the evolution, over te centuries, of the ratio of the cost of transporting 1 tonne for 1 km in auxiliary and major transport, on side and main roads:

paths and dirt roads (power of humans and animals)	1.5:1
dirt roads and paved, not upgraded roads (horse transport)	1.5:1
paved, not upgraded roads and upgraded paved roads (car transport)	1.3:1
upgraded paved roads and rail lines (car and rail transport)	1.3:1

Source: [94], [66], [10], [31] and the author's calculations.

This observation is of great importance to explaining the system of roads and their location with respect to railways. It provides an explanation why the relations between the directions of side and main roads (of various degrees of importance) are in fact identical, in the same categories even if they emerged in different Times, in different technical and economic conditions. The obseravtion also has some methodological value. By paraphrasing Ch. Lyell's „uniformanitarian" method introduced to geology, one can say that the stability of the ratio of costs of the forms of transport leads to a conclusion that transport complexes have grown historically. However, what need to be considered is a small drop in the cost ratio reflected in modern transport. While the identified values of measures are very similar, the drop must not be viewed as incidental because it has already left its mark on the structure of complexes of transport networks.

4. Differentiation of geographic and economic space

The goal of this chapter is not to present the traditional relations between the geographic environment and the economy on the one hand and with transport on the other. From a large group of issues, I have selected one: i sit possible to capture the influence of the geographic environment and economy in a theoretical complex of transport networks? In fact, this issue comes down to the following question: how far can we go in generalising the impact of the geographic environment and economy on the emergence of and the form of complexes of transport networks?

As for the geographic environment, two types of generalisations have taken place to date: systematic and typological.

Within the systematic generalisations, an attempt has been made to explain the impact of the specific components of the environment (the genological structure, terrain, waters, the climate, soil, the fauna and flora) on the development and distribution of the transport network, regardless of where the transport was arranged. For example, the value of the different genological formations has been evaluated as the bed for transpsort routes and construction materials together with the resistance of the various inclinations of the land, the acceptable deadweight tonnage of ships at various guaranteed river depths, the conditions for snowdrifts, sand drifts, avalanches and debris flows, fog to form that either facilitate or hamper the impact of various plants, the power of pack animals and beasts of burden. In research into transport networks, of importance are the differences in specific components' impact on the various types of transport.

Typological generalisations were intended to highlight the natural spatial units which are of significance to transport. With this in mind, transport-related regionalisation was carried out on the geographic environment on the continents, zones, provinces etc. Typically, the procedurę would be limited to one criterion, that of morphology. Application of another criterion, e.g. the flora, led to new regionalisation (E. Otremba) i.e. to two single-feature regionalisations instead of one two-feature regionalisation. In fact, morphological differentiation represents the leading trait in the diversity of the landscape, esepcially within the same climatic zones. Therefore, the latest attempts at landscape typology and taxonomy of regions are still derived from the division of the geographic environment into lowland, upland and mountain landscapes. The remaining components of the landscape are only taken into consideration when addressing lower-rank units. However, on a micro-regional scale, the dependence of the local climatic, water, biotic and soil conditions on terrain, inseparably tied with the nature of the rockbed, is even greater. Transport also tends to be very sensitive to terrain because of the difficulties accompanying the inclination of the land. This sensitivity is gauged by the biggest inclinations and the smallest radii of the arcs acceptable in road construction in different types of transport (Table 4).

Table 4

Type of transport	Biggest inclinations	Smallest radii (m)
1. Railways		
main	1 : 40	300
side	1 : 25	200
2. Roads		
main	1 : 25	300
side	1 : 15	50
3. Waterways	1 : 1200	300

Source: C. Pirath [85].

A railway line's inclination exceeding 3‰ decreases the locomotive's tractive force by half; inclination exceeding 10‰ (not unusual in the mountains) decreases the locomotive's tractive force by 90% while inclination of 15‰ makes a locomotive move by itself (on adhesive sections; sections of rack railways and cable transport can be much steeper). As the inclination grows, so do the costs of using railways. Covering 1 km of a section with inclination of 20‰ costs as much as covering 2 km on a plain [39]. Roads are less sensitive to the inclination of the land. Therefore, in the mountains roads are marked out differently than railways. Typically, roads reach the end of a transverse valley and climb all the way up to the most convenient mountain pass or they start to climb gradually the slopes from they very beginning of a mountain range. On the other hand, railways owners choose the shortest routes by boring tunnels. Oftentimes, a passage from lowlands to the middle storeys of mountains and up to high mountains requires a change of the means of transport. These changes contribute to obstructing movement of goods and passengers and, as a result, emergence of estates. Lowlands host heavy railways, motorways and navigable rivers. On the medium levels, railways are less common while the importance of lighter types of roads grows, inland navigation takes place only on the outskirts and in good hydrographic conditions at that. In high mountains, roads are also exceptions, the role of paths taken by animals and humans carrying goods grows while navigation of only local importance is limited to sporadic lakes. The biggers the transport needs of the lines crossing mountainous areas, the more blurred the technical differences in the equipment related to the increase in relative and absolute height; however, they rarely disappear completely. In the case of railways, the differences remain in the gauge of the rail tracks (narrow-gauge railway in the mountains) and, typically, in the length of trains, the type of the locomotive etc. The Andean railways, built in high altitudes, are a work of engineering art (to which Polish engineers,: E. Malinowski and W. Folkierski, made a contribution). The rail line between Lima and Cerro de Pasco goes up to **4,840** m APSL in Galera tunel; the line between Mollendo and Puno climbs up to **4.470** m APSL on the Cruzero Alto Plateau; the line between Ascotan and Collahuasi runs at **4,820** m APSL, the Rio Mulato line goes to Potosi at **4,880** m APSL.

In lowlands, the environmental forces counteracting emergence of regular systems of transport networks, are relatively weaker. Lowland countries, especially the ones surrounded by mountains or by a sea and mountains, are typically regular as well as homogenous and centralised in the capital city. A case in point is France, Hungary, the European part of the USSR. Rivers – whether they divide or unite – impose deviations from both major and monor directions. The tendency of roads to run toward river mouths, fords and to be located in river valleys, is reflected in the diminishing values of the gradient in parallelisation. The deviating impact of rivers grows in uplands and mountains. In generalo, in old mountains tyhe impact is positive (attracting transport) while in young mountains it is negative. The vast ranges of the Rhenish Massif would be a major transport obstacle if the Rhein did not cross it in the middle and, on top of an excellent waterway, did not provide topographic conditions for constructing roads and railways. Even today, traffic in the Rhine valley is extremely heavy while in its vicinity, on the hills, it is minimum. Young mountains with open and narrow canyons do not offer good road or rail routes; instead, they pose the threat of flooding. For this reason, roads tended to go round canyons and would run along drainage divides.

It was owing to modern technology that these roads were available to mass transport (e.g. the upper Reuss valley).

Mountains not only have a transport network different from lowlands; it also varies from one type of mountains to another (terrain, bedrock). High, compact mountains and large transverse valleys have a network different from that in losely structured mountains divided into many chains (the Jurassic) where long valleys are connected through short transverse valleys. The network is still different in fault blocks. In mountains built from impermeable crystalline structures, the network is formed differently than in mountains consisting of limestone and sandstone where the water groove valleys that are narrow and hard to cross. A very case in point are the differences in the road network in the Ore Mountains and the adjacent limestone Saxon Switzerland.

In a mountainous transport network, side roads are underdeveloped (they only represent single tentacles toward the hills) while peripheral roads have not emerged at all. In mountains with easy passes, sometimes many main roads converge. In North America, even if the Atlantic coast from Boston to Norfolk witnessed mass colonisation, the main roads running to the west concentrated in a relatively short section between New York and Baltimore as a result of the impeding impact of the Appalachians.

The routes of various types of transport which emerged in the same physico-geographic regions can differently deviate from the regular system. Even when railways connect the same destinations, they typically run along different routes and have intermediate points different from the old roads (only some narrow-gauge railways run along the edge of roads). In this case, prosperous cities ignored by railways tend to decline. On the other hand, villages located on the route of railways, grow into towns. Typically, railways have better suited routes than the older roads. This is because, resulting from limited technical possibilities, attempts were made in the past to maximally use terrain which ocercomplicated the course of roads. Now, owing to technical progress, it is possible to sstraighten roads; this is done to facilitate car traffic and to minimise the lossess incurred by a growing numer of vehicles passing every deviation of a road. Straightening of roads also takes place in the railway network. A case in point is the connection between Bologne and Florence; it was shortend by means of boring deeper tunnels. In general, the more important a tranport route (the bigger the freight) the straighter the route should be.

On top of the vertical arrangement, O. Blum takes into consideration the horizontal arrangement and tries to define the types of transport location. He differentiates [11] between: 1. foothold location (Phoenicia, te netherlands), 2. platform location (Schleswig, Panama), 3. central location (the Saône basin, Lake Lucerne area), 4. basin location (Hungary, Mesopotamia), 5. Mountainous location: a. ridges (Tyrol), b. nests (Ethiopia), c. slopes (Chile), 6. Artery location (Egypt, Siberia), 7. location at inner seas (Roman Empire, Japan), 8. Location on islands and peninsulas.

While the geographic environment does not determine transport, its types must not be disregarded in identifying typical complexes of transport networks. Typological generalizations about the impact of the environment, while very important in geographic research, are not always sufficient. When making attempts of making a theoretical model of a complex of transport networks, the effort should be targeted at higher degrees of generality. It seems possible to achieve by means of mathematical methods. Unfortunately, wse are still far from satisfactory solutions.

In algebraic models, the impact of the environment may be presented by means of adequately selected parameters which express the resistance put up by the environment to transport. Since our aim are complexes of transport networks which fulfil the prerequisite of minimum construction and operation costs, the resistance would be expressed in economic value categories. I could use here the concept of virtual length as defined by W. Launhardt [56]. The virtual length of a specific, diverse-profile transport line indicates the length of an arbitrary transport line, straight and horizontal, that transport of 1 ton costs as much as on a specific line. Let us assume that the specific line amounts to 100 km and transport of 1 ton along the entire length costs Pln 80.00. At the same cost, along a straight and horizontal line, 1 ton can be transported for a bigger distance, e.g. 150 km. The ratio between virtual length and actual length (in this case 1.5 [150:100]) is referred to as the virtuality ratio. Its values, established in an experimental way, could be introduce to equations as parameters expressing the structure of complexes of transport networks.

Far-reaching generalizations of geometric models seems possible owing to topological methods. K. Dziewoński [29] drew attention to the need of applying topology concepts in a geographic analysis. He emphasized the topological equivalence and economic space and a possibility of mutal transformation thereof. However, as for transport phenomena which are of linear nature (one-dimensional), this possibility looks different than for surface (two-dimensional) phenomena. While mutual projection is possible – unambiguous planes on a straight line (because a set of all the points on a plane have the same cardinal number) and a set of all points in a straight line, this correspondence is not topological because the requirements of continuity are not satisfied. Indeed, figures of different dimensions cannot be topologically equivalent.

The relations between the theory of stationary points and the typology concepts has been indicated before, e.g. in the course of looking for passages in the mountains. From a mathematical point of view, this search comes down to a mini-maximum task with a goal of finding mini-maximum stationary points of function of two variables $f(x, y)$. These points have been named saddle points which, in a mountainous landscape can be pictured as passes. In transport interpretation, the solution shows the route whose highest point is possibly low (the lowest maximum).

Transport roads, which are elements of complexes of networks, have properties depending on directions. These properties are called anisotropic by analogy with anisotropy of the physical properties of materials. The biggest discrepancy is between the properties of transport roads running in main directions and transport roads running in minor directions.

Anisotropy is a feature of complexes of transport networks even in areas least diverse with respect to physical and geographical properties. Anisotropy results mainly from the requirements of transport itself and anisotropy of economic space. There is a dependence between anisotropic properties of complexes and economic space. Research into anisotropy should contribute to confirming the argument that transport is one-dimensional and the number of roads is finite; it would also prove useful in reducing in geographic and economic models the simplifying assumption that transport is available everywhere.

Some important anisotropic properties of economic space can be largely generalised. They include primarily concentration of production, population and settlements along transport roads, especially along main roads. When transport goods are concentrated in major directions, the number of transports decreases and expensive road investments which dacilitate transport are justified. This property is therefore conditioned by large scale benefits, in production and transport alike. In rural areas where dispersed transport prevails, the cost of 1 tkm and 1 pkm is twice to three times higher than in industrial areas with concentrated transport ensuring intensive use of means of transport. C. Pirath calculated that in Württemberg, industry dispersion, exceeding the optimum level, resulted in an increase in transport costs by 10% [85]. However, when the concentration of transports exceeds a certain level, transport fixed assets grow considerably.

Concentration of production, population and settlements along transport roads of various importance can be measured statistically and the dependences can be expressed quantitatively. For example, when A. Lösch discussed concentration of central locations (central functions) he assumed that their number on primary lines is 2.5 times bigger and on secondary lines approximately 1.5 times bigger than on tertiary roads. W. Isard noted that in an area ranging along a transport line, farming may be more intensive and structurally different than farming in an area located closer to human settlements but with poor transport access. Deviations of a settlement network caused by transport repeat themselves with a regularity which others already tried to capture in theoretical patterns (J. Cybulski, K. Lichtenstein and T. Rutti) [27].

The diversity of the spatial structure of complexes of transport networks is strictly related to the uneven concentration of economic space. Previously, emphasis was only placed on the diversity of the number (density) and importance (hierarchy) of roads in complexes which emerged in different geographic and economic conditions. Changes to the shapes of complexes in transition from equally dense space to anisotropic space have not been studied.

The changes resulting from increasing the importance of one direction in a complex of transport networks are quite telling. It is externally express by raising the technical standard of a road running in that direction.

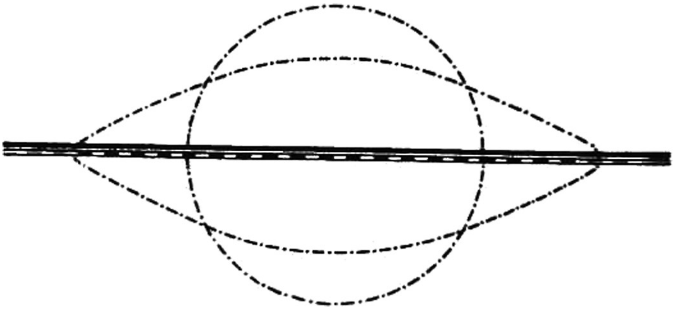


Fig. 23. Reorientation of a (service) market area following transport enhancement

An enhanced road (let's assume that it has become a major road in the process) reorients the (service) market area. Other things being equal, these areas sort of extend and flatten along the higher standard road. This is presented in Figure 23. This reorientation can be presented by means of mathematical functions. I suggest to adopt function $K = \psi \cos \alpha + c$ as a mathematical approximation of the changed shape of market areas where K indicates the shape of a market area, ψ – the proportionality coefficient, c – a constant independent of the direction[31]. The ψ coefficient needs to be determined experimentally. Wherever its value decreases, the shape of the function increasingly flattens. This indicates an increase in the range of costs of transport by an enhanced and unenhanced road. Therefore, there is a negative correlation between the value of the coefficient and the cost ratio.

However, estates emerge more frequently and tend to be larger along an enhanced road. As a result, their respective market areas expand in

---

[31] Market areas as variables are presented in detail by W. Isard [43]. He ponders primarily the issues of the limit separating areas supplied by two producers. The limit equation has the following form:

$$\sigma_\rho + r_\rho s_\rho^o + \sum_{i=A}^{F} b_i r_i s_{i\rho} = \sigma_v + r_v s_v^o + \sum_{i=A}^{F} b_i r_i s_{iv} \qquad (\rho \neq v) \ (\rho, v = 1, \dots \eta),$$

where σ_ρ, σ_v indicates the final production costs in locations P_ρ and P_v, r – the tariff for a product unit, s_ρ^o, s_v^o – the distances from P_ρ to P_v any point on a shared border, $A...F$ – various raw materials used in production, b_i – a constant coefficient indicating the numer of raw material units i used per product unit.

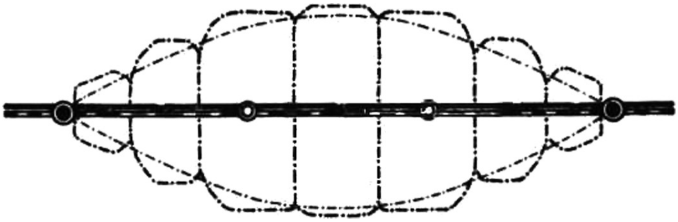


Fig. 24. A network of markets along a main (enhanced) road

a crosswise direction and get narrower in a longitudinal direction. The elongated shape remains typical of a network of markets ranging along the enhanced road (Figure 24).

These observations provide a general overview in the directions in which minor roads strain as a result of an emergence of main and enhanced roads: 1. minor access roads, connecting the more densely located production places with the auxiliary facilities, deviate towards a perpendicuar location; 2. Minor peripheral roads are flattened.

The degree to which minor peripheral roads deviate when the properties of directions change, can be established by adopting transport costs affected by directions as the measurement of anisotropy. Adoption of average diversity seems erroneous because on an enhanced main road with concentrated transport (e.g. electric-powered trains), unit costs are

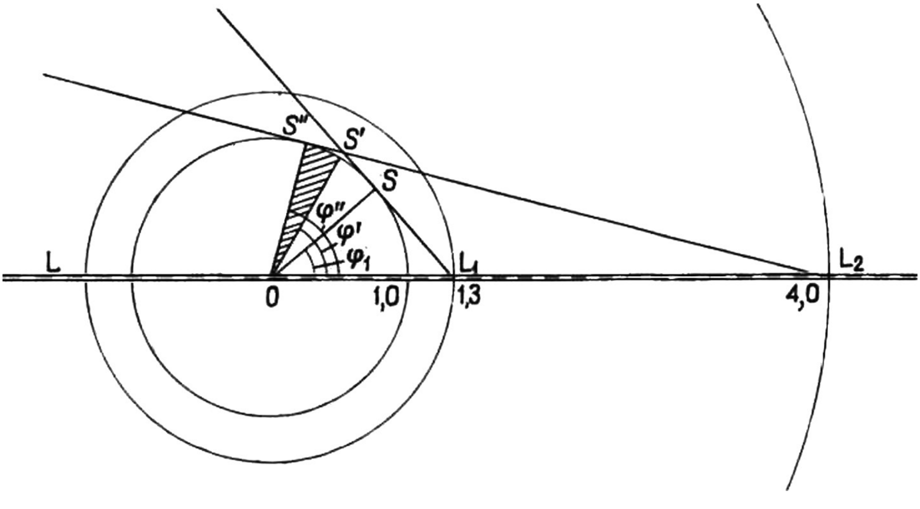


Fig. 25. Deviation of a minor road in the anisotropic complex of transport lines. The impact of the main (upgraded) road

lower than the average costs in a network of roads of a specific type of transport. The situation is the opposite on a minor road (e.g. a cobbled access road). Therefore, the agreed ratio of train and car costs (1:1⅓) needs to be changed by raising it to approximately 1:4 (provided that the reloading costs are considered). The related change of location of a minor road against a major road is presented in Figure 25. The inner circle represents an area homogenous with respect to transport. Let's assume that line LL_2 is the main road with relatively lower costs of transport. From the inside of the inner circle we draw two auxiliary circles with radiuses 1⅓ and 4.0 i.e. representing the previously agreed costs. From points L_1 and L_2 we draw tangents to the inner circle in points S and S''; from the centre, we draw radiuses OS and OS''. The radiuses can represent minor roads. The gradient of road OS against the main road LL_2 amounts to approximately 40° i.e. as much as in the elementary set as presented in Figure 21. The difference between the gradient of road OS and road OS'', depreciated by the angle of the reloading costs *(SOS')* is the sought deviation of the minor road related to the factor of anisotropy (75° – 61° = 14°).

In comparison with a single road, determining the deformation of a complex of minor roads is more complicated. In my opinion, tensors seem to be the desired method while physical crystallography can provide an analogy [79].

Let me review the north-eastern sector of the network in a rectangular arrangement. We need to find out the deformation of the rectangle (a square used in the example) in point A depicting a transport hub (Figure 26). The coordinates of point A with reference to the axes of the complex amount to (x_1, x_2). First, we define the deformation of the section as a result of transfer: it is a ratio of the increment of length and the original length. In a general case it has the following form:

$$e = \lim_{\Delta \to 0} \frac{\Delta u}{\Delta x} = \frac{du}{dx}.$$

In a two-dimensional space, the transfer is expressed by these two equations:

$$\Delta u_1 = \frac{\partial u_1}{\partial x_1} \Delta x_1 + \frac{\partial u_1}{\partial x_2} \Delta x_2,$$

$$\Delta u_2 = \frac{\partial u_2}{\partial x_1}\Delta x_1 + \frac{\partial u_2}{\partial x_2}\Delta x_2.$$

In short: $\Delta u_i = \frac{\partial u_i}{\partial x_j}\Delta x_j = e_{ij}\Delta x_j \quad (i, j = 1, 2)$.

Because $[\Delta u_i]$ and $[\Delta x_j]$ are vectors, $[e_{ij}]$ is a tensor.

Let consider vector *AB*. After replacing it with $\Delta x_2 = 0$ we can write:

$$\Delta u_1 = \frac{\partial u_1}{\partial x_1}\Delta x_1 = e_{11}\Delta x_1,$$

$$\Delta u_2 = \frac{\partial u_2}{\partial x_1}\Delta x_1 = e_{21}\Delta x_1.$$

The geometric interpretation of vectors Δu_1 and Δu_2 is as follows: e_{11} represents the extension of vector *AB* towards axis Ox_1 (tensile deformation).

$\lim_{\Delta\to 0}\frac{\Delta u_1}{\Delta x_1} = \frac{\partial u_1}{\partial x_1} = e_{11} \cdot e_{21}$ presents anti-clockwise rotation of vector *AB* (shear deformation). It is measured by $\mathrm{tg}\,\vartheta = \frac{\Delta u_2}{\Delta x_1 + \Delta u_1}$. Similarly, deformation of vector *AD* (e_{22}, e_{12}) can be calculated.

Clearly, the shift $\vec{u}$ transforms the square arrangements of sections *ABCD* into a parallelogram *A'B'C'D'*, changing not only the length of the

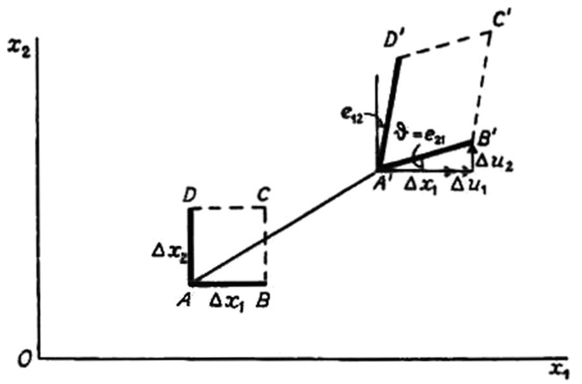


Fig. 26. Deformation of a complex of minor roads (the north-eastern sector)

sections but also the angles between them. The reduction of the angle between the bolded sections is correct. It can be attributed to a relatively bigger concentration of the mass waiting to be transported. The theorem at play here is the increase in transport mass as the angle between the directions of roads increases[32]. The linear relation between the deformation tensor e_{ij} and the scalar – the mass waiting for transport Q can be expressed in an equation $|| e_{ij} || = || a_{ij} || \Delta Q$ where the tensor a_{ij} is a coefficient defining the deformation of a system of minor roads as the transported mass grows by a unit. However, only a search for correlational rather functional relations has practical sense.

---

[32] This theorem was proven by Z. Wasiutyński [104].

IV. A complex of transport networks: a model

Attempts at depicting a complex of transport networks in a theoretical and synthetic way should take into account the structure of the complex' models. One way of accomplishing it is adapting the models constructed for other issues. These models (beside the models mentioned in chapter II) include: 1. a travelling salesman model, 2. a copper-centre model, 3. a centralised network model. The first one requires adaptation which involves a hierarchy of routes in a complex. In the original version, routes are of equal importance. Therefore, first we need to set a system of the main routes, followed by a system of secondary, tertiary etc. routes. The supplemented model has an important drawback: the routes are not sufficiently cohesive. The model's geometric picture is reminiscent of H. Steinhaus' dendrites. The benefit of the second model is the fact that it contains variables in proportion to the costs per unit of road distance. However, it does not represent a complete complex and does not take into account an already existing regional centre or factors other than minimisation of economic distances which define the centre's location. This may be applied in small territorial units, especially new investments where a centre's location is a key variable. When a centre is known the shape of a centralised network connecting it with the settlements on the service area represent the third model.

An in-depth analysis of all the three models indicates a need of a more adequate model of a complex of transport networks. Following this line of reasoning, I make an attempt to make assumptions, to define the fundamental properties and partial solutions offered by the model. Its name, the model anisotropic, highlights the properties that have not been sufficiently examined and which are very interesting from the point of view of economic geography.

1. The travelling salesman model

Definition of the problem: a travelling salesman sets off from a type p settlement and intends to visit all the n settlements of this type in the region (the reasoning can be extended by including other types of settlements). Which route of the salesman[33] will be the shortest?

The number of routes (activities) that can be arranged [65] between the n settlements, if the initial settlement is fixed, amounts to $n - 1!$ The number grows extremely fast with the number n, e.g. $20! =$ 2,432,902,008,176,640,000. Therefore, it would be difficult to establish the acceptable routes, even for computers.

However, in mathematics, when an issue is made more complicated, they become more suitable for analysis. It is easy to demonstrate that a finite set F can be extended to an infinite set F^* and a set of indices subordinated to the specific operations extended to a new set so that the extended problem becomes a problem of linear programming. Its solution is one of the operations from set F. On the other hand, each problem of linear propagation can be treated as a zero-one game. Therefore, the problem at hand is an example of the unexpected applications of the game theory.

The linear programme is copied from H.W. Kuhn [52]. Let's assume that T_n is a set of routes defined by n x n of matrix permutation $t = (t_{ij})$. Let's assume that C_n represents a convex edge T_n w n^2 – Euclid's dimensioned space. Polyhedron C_n extends $(n^{2-3}n + 1)$ – the dimensional linear variety of all the $x = (x_{ij})$ z $x_{ii} = 0$ and $\Sigma_j x_{ij}$ for all the i and j. Each non-zero matrix $b = (b_{ij})$ defines the half-space $\Sigma b_{ij} x_{ij} \geq \beta$, supporting C_n in line with the rule that $\beta =$ minimum $\Sigma b_{ij} t_{ij}$ for $t = (t_{ij}) \in T_n$. All the C_n walls i.e. its $(n^2 - 3n)$ – dimensional sections with a supporting hyper plane, results from a non-negative integral b.

---

[33] When applying the travelling salesman's model to designing a complex of roads, it is recommendable to imagine that a salesman travels by airplane and can land in any settlement.

2. The copper-centre model

The copper centre of a telephone network [68] is a name for the location of a telephone switch where its connection with the concentrators where the wires from specific telephones meet; it requires the smallest amount of copper wire. Instead of presenting cables of varied gauge (depending on the number and thickness of the wires) I can apply this rule to roads with diverse construction costs (depending on the number of lanes, the quality of the surface etc.) and transported goods. Point Q is the copper centre when

$$\Phi(Q) = \sum_{i=1}^{n} c_i r_i = min,$$

where c_i indicates a positive number proportional to the costs of the length of the cable connecting the switch Q with the concentrator P_i, and r_i indicates the length of the cable QP_i. The minimum of the function $\Phi(Q)$ can be identified by means of three methods. The simplest solution is a dynamic analogy referred to as the location theory [82]. It turns out that the function $\Phi(Q)$ whose minimum I intend to determine equals the potential of a specific field of powers. When points P_i are not all located on a straight line which in practice is a rule, $z = \Phi(x,y)$ represents a surface convex to the bottom. This suggests that there is exactly one minimum of function $\Phi(Q)$. The second method of identifying a copper centre is based on the convexity of potential. The method relies on applying polygons to limit an area that needs to host the copper centre. A finite number of steps results in an area small enough to compensate for an error made by adopting any point rather than an exact copper centre. The third method comes down to iteration based on specific equations.

3. The centralised network model

We know the area and the limit, the location of the centre to which all the transport are oriented, the number of roads (*m)* and their final points R on the verge of the area, the access roads are parallel to each other. We need to find the shape (form) of a network of roads, ensuring a minimum of total transport costs. This issue is most frequent in urban planning which has also provided the solution, based on the calculus of variations [34].

Let's assume that $S_h(x)$ means roads, e_h means the limits of their service areas ($h = 1, 2,..., m$) and m – the number of roads. In a mathematical approach, we should select an arrangement from among all the functions $S_1, S_2..., S_m, e_1, e_2..., e_m$ that will minimise the costs borne by transport within a year (I). So

$$I = \int f\big(x, e_h(x), S_h(x), S'_h(x)\big)dx = min.$$

It is good to start the search for an optimum shape of a network of roads by outlining the optimum course of a single road, temporarily assuming a fixed service area.

For h – the area, the annual investments amount to:

$$\mathrm{I} = A_{uII} + \int_{x=0}^{i} \Big\{a_w \Big[\int_{e_{h-1}(x)}^{S_h(x)} q(x,y)\,(S_h(x) - y)dy + \int_{S_h(x)}^{e_h(x)} q(x,y)$$

$$\big(y - S_h(x)\big)dy\Big] \; + \Big[a_{uI} + a_w \int_{\xi=0}^{x} \Big(\int_{e_{h-1}(x)}^{e_h(\xi)} q(\xi, y)dy\Big)\, d\xi\Big] \sqrt{1 + S'^2_h(x)}\Big\}\, dx,$$

while for a sum of partial areas $\mathrm{I} = \sum_{h=1}^{m} \mathrm{I}_\mathrm{h}$,
where A_{uII} – costs independent from the traffic, related to access roads in the entire area (Pln/year), a_{uI} – costs independent from the traffic, related to the sought roads (not access roads), calculated per unit of distance (Pln/km in a year), a_w – unit costs directly dependent from the traffic (Pln/tkm), q – density of the transported mass (t/km^2 in a year).

Beside the functions $Sh(x)$ the unknown includes the functions $e_h(x)$ which, however, can be eliminated. They can be expressed as functions of the curves $S_h(x)$ and $S_{h+1}(x)$ i.e. the adjacent roads, separated in the range of their service by the function curve $e_h(x)$.

$$e_h(x) = e_h(S_h, D_{h+1})$$

If we assume an even distribution of the sources of traffic and a regular structure of a network of roads, the limit between two adjacent areas stems from this equation:

$$e_h(S_h, S_{h+1}) = \frac{1}{2}\left(S_h(x) + S_{h+1}(x) + \int_x^i \left(\sqrt{1 + S'^2_{h+1}(x)} - \sqrt{1 + S'^2_h(x)} \right) dx \right).$$

A variant derivative of an integral (*I*) determines the shape of the network of roads, fulfilling the condition of minimum transport costs. The solution is provided in these differential equations:

$$[f]S_h = \frac{\partial f}{\partial S_h} - \frac{d}{dx}\frac{\partial f}{\partial S'_h} = 0.$$

The sought shape of the network stem from the *m* of the equations i.e. the number of roads.

4. The anisotropic model

The morphology (the shape, the structure) of the anisotropic complex of transport networks is a function of the hierarchy, the location and dimensions of roads for different types of transport which represent a complex. Therefore:

$$M_z = f(\chi, \varphi, \zeta)$$

where M_z means the complex' morphology, χ – the hierarchy, φ – the location against each other, ζ – the dimensions of the roads in the complex. The dependent variables are calculated by means of the transport load (tkm/km), the angles, the ratio of the number of roads in hubs and the ratio of the length of the roads in a complex.

Let's assume that the optimum criterion (the selection function) is as follows:

$$C^A_{S+E} = \sum_{i=1}^{n} S_i c_i^S + \sum_{i=1}^{n} \Phi_i l_i c_i^e = min \qquad (i = 1, 2, \ldots, n).$$

Here, $C^A{}_{S+E}$ represents the general cost of the anisotropic complex which consists of the costs of building roads and the costs of transporting goods (Pln), S_i – indicates the distance of the roads for z-type of transport (km), $c^s{}_i$ – the unit cost of building roads for i-type of transport (Pln/km), Φ_i – the transported goods in the i-type of transport (tons), l_i – the average transport distance in the i-type of transport (km), c^{e}_{i} – the unit cost of transporting goods in the i-type of transport (Pln/tkm). Therefore, an anisotropic complex should fulfil the following condition:

$$C^{A}_{S+E} < C^{U}_{S+E},$$

i.e. it should operate at costs lower than the costs of the complex that can be built on the basis of the traditional models of a transport network (cf. the model of a simplified complex). The division of costs into the costs of building roads and the costs of transporting goods results from their incompatible impact on the morphology of an anisotropic complex. It is possible to build a complex that is most economical with respect to the costs of building roads but is not the most economical when it comes to transporting goods and vice versa.

The conditions are defined in the following equations and inequalities:

$$1)\ \Phi_1^S > \Phi_2^S,$$

$$2)\ \varphi \leq 180°,$$

$$3)\ S_1 < S_2,\ \ S_1^W \leq S_2^W,$$

$$4)\ c_1^S > c_2^S,\ \ c_1^e < c_2^e,\ \ c_1^0 < c_2,$$

$$5)\ \Delta c_S < \Delta c_e,$$

$$6)\ c_1^0 : c_2^0 = const,$$

$$7)\ c\varphi <\ c_2^0 \vee c\varphi > c_2^0, c\varphi < c_2^0 \supset \mathrm{K}, c\varphi > c_2^0 \supset \mathrm{S}$$[34],

$$8)\ \Phi_2^x = I_2^\omega,$$

$$9)\ G_1 > G_2 < G'_1.$$

Here, Φ^{S}_{1}, Φ^{S}_{2} means the traffic on the main (primary) road and the side (secondary) road (tkm/km), S^{w}_{1}, S^{w}_{2} – the number of main and side roads in a hub, c^{0}_{1}, c^{0}_{2} – the total unit costs of transport by main and side

---

[34] V and ⊃ – symbols of the alternative and implication from the logic of sentences (Peano-Russell's symbols).

road (Pln/tkm), Δc_s, Δc_e – incremental costs of building roads and transporting goods (Pln/km), the condition [7] – the unit cost of multimodal transport ($c\varphi$) can be lower or higher than the direct costs of transport on a side road; if it is lower, the main and side roads are complementary (K); if it is higher, the side road substitutes the main road (S), $\Phi^x{}_1$, $\Phi^\omega{}_1$ – access traffic (transport over small distances) and direct traffic (transport over medium and far distances) on side roads (tkm), the condition [9] – the density of transported goods in a sector contained between two subsequent main roads: $G_1(G'_1)$ – density in an area adjacent to the main road, G_2 – density in a transition zone served by side roads.

4.1. Hierarchyofroads

The importance of roads, their role in a complex of transport networks, is measured by the size of the transport. In a theoretical model, only potential transports are considered. To evaluate them, the gravity model is employed [44]. However, in a pure form, the picture is very simplified. Adjustment to potential transports requires introduction of the respective weight to the gravity model

$$F_{ij} = \psi \frac{d_i(L_i)^\alpha \cdot d_j(L_j)^\beta}{l_{ij}^\eta},$$

where F_{ij} means the power of transport attraction between centres i and j, ψ means regular proportionality, L_i, L_j – the population, d_i, d_j – the population's income (for passenger transport) or tonnage of industrial production (for goods transport) per 1 inhabitant, α, β – exponents of the inhabitants' mobility[35] or transport of industrial production, l_{ij} – the distance between centres i and j [36]. In order to establish potential transports on a specific road, we need to add the powers of transport attraction between the n centres located on the road's local and transit service

---

[35] The exponent's value grows as the size of an agglomeration grows.

[36] W. Mylroie who examined passenger transport between settlements concluded that the correlation with the results of theoretical calculations is relatively closest when in the gravity model $\alpha = \beta = 1/2 = \eta = 2$, $d_i = d_j = 1$ [44].

areas (sum of $\frac{n(n-1)}{2}$ operations). By limiting the service area, one can calculate the potential transports on the subsequent sections of the road.

As the distance from the main settlement grows, the following equation captures better the change in the importance of the roads sections:

$$\log \Phi^{v}{}_{i} = \psi \log l,$$

where $\Phi^{v}{}_{i}$ indicates the potential traffic on an i-road, ψ means the proportionality coefficient, l is the distance in km. D. Neft [77] estimated empirically the value of the proportionality coefficients for New York, London and Paris (passenger traffic). They are contained within the following brackets: 3.27–1.64, 3.14–1.26 and 2.84–1.36. With these values, the correlation coefficient of the two variables amounted to 0.69, 0.82 and 0.83.

Most frequently, the importance ratio of various roads is reflected as a ratio between the main road and a side road. Are there relevant correct correlations? I have carried out an experiment of comparing the transport sizes (in tkm) on main and side roads. I compared separately transports on each main road and the sum of transports on the related side roads. In the experimental region, the main road tended to prevail. Because a large part of transports from side roads do not reach the main road or end in the junction settlements, the advantage of the main road stems from the considerable power of own and transit sources of traffic (production benefit and large-scale transport). It can be written down as follows:

$$\Phi_1^s > \sum_{j=1}^{n} \Phi_{2\,j}^s.$$

When the experiment, initially limited to the relation between the main road and side roads, the main railway and side railways, was expanded by the relation between the rail and distribution and access roads, the advantage of the main road (rail) was multiplied.

As a result of the diversification of traffic on the main roads to parts dependent on and (relatively) independent from the traffic on side roads, the mutual reactions of both categories of the roads to changes in traffic are not always instantaneous. The traffic on the main road can grow, without a distinct increase on side roads. Ultimately, the continued revival of the main road triggers off transport activity on side roads. In general, the longer the main road the greater its impact on side roads. This is because the range of profitable transport grows as the transport distance increases on the less expensive road (the main road). Traffic can also be generated by starting with the side roads (farming regions). The related increase in traffic on the main road can be presented by the following formula:

$$\Phi_1^s(t+1) = \Phi_1^s(t) + \sum_{j=1}^{n} \varepsilon_{12_j} \Phi_{2_j}^s(t+1),$$

with

$$\varepsilon_{12_h} = \frac{\partial \Phi_1^s(t+1)}{\partial \Phi_{2_h}^s},$$

where t indicates year t, $t+1$ – the following year, ε_{12_j} – increase in traffic on the main road as a result of increment by a unit of traffic on j-side road, ε_{12_h} – the impact of growth in traffic on h-side road on increased traffic on the main road.

The hierarchical structure of the complex is so that lower-grade roads duplicate higher-rank roads in the more important directions and supplement in less important directions, in accordance with the substance (parallelization) and complementarity law. All sorts of transport: waterways, railways and roads run in the most important direction (a complex' axis), assuming favourable geographic conditions. In the secondary directions run railways and the main roads. The tertiary directions are served by a network of side roads. The latter can be further divided into distribution roads, vicinal and capillary roads. The enhancing transport effectiveness sometimes requires upgrading a complex' hierarchy. On the other hand, the trends to create a transport system adjusted to operations in changing conditions curtail the growing hierarchy (antinomy)[37].

---

[37] It would be interesting to find out if propositional calculus is useful in determining whether a complex is obstructed or not with different states of the

Therefore, a complex of transport networks is created by superimposing networks of roads dedicated to different means of transport. If economic space were homogenous or regularly dense, each network separately and all of them jointly would comply with the condition of minimum transport costs for a regular hexagonal (extended into an equilateral triangle) or square arrangement. However, because economic space is anisotropic, the complex' optimum shapes deviate from these arrangements. In which direction?

4.2. The location of roads against each other

It is determined by the entire presentation above and, first and foremost, the fact of splitting the transport process into direct transport and multi-modal transport, coupled by re-orientation to the market areas affected by transport enhancement.

In multi-modal transport, taking place on side and main roads, side access roads deviate towards a perpendicular location against the main roads. As a result, the distances of the more expensive access traffic as well as traffic which ends in touch-point settlements, will be maintained within the optimum (according to the law of refraction, transport with corrections). On the other hand, direct transport which is based on the shortest connections, leads to sharpening the angles and, consequently, flattening the regular (hexagonal and trigonal) complex of peripheral side roads towards the main road. However, for the direct roads to be profitable, the traffic needs to be sufficiently big. It has been assumed that the traffic equals the access traffic. In countries with well-developed automotive industry, the state of equilibrium has been achieved or surpassed (cf. Table 2) while the change in the scope of rail work and the growing share of sidings in rail transport is an indication that the growth in direct car transport has not ended yet. Therefore, the flattening may continue.

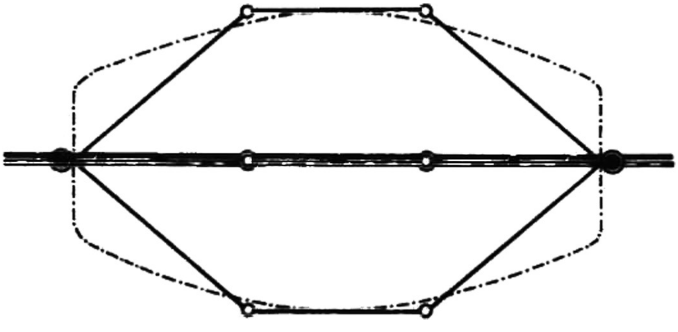
The split of the communication process takes place within limits delineated by the an arrangement of market areas and their centres, an ar-

---

transported mass or in studies of transported mass flows. To date, it has been applied to solve a similar problem in the area of electrical grids.

rangement affected by transport. A special case of mutual impacts takes place when enhancing of transport causes re-orientation of the market areas which, in turn, defines the flattening of the complex of side roads along the main road. In my opinion, the function $K = \psi \cos a + c$, defining the re-orientation of market areas could also be the first mathematical estimate of the flattening of a complex of side roads. For the resultant convexity to be aligned with the distribution of the centres rather than the course of the limits of the market areas, the value of the invariable c needs to be raised and the coefficient y carefully selected. While the resulting geometric picture would be a figure reminiscent of an ellipse rather than a flattened hexagonal complex, this does not mean that, compared against the actual complex, the deviation would be bigger in each case. Besides, Figure 27 shows that the differences between the graph of the function $K = \psi \cos a + c$ and a flattened hexagon are not very large. (The straight sections connect settlements disctributed as in Figure 24; $\psi = 1$; $c = 1$ cm).

Side direct roads run along the lines dividing the service area into a sub-area of profitability of direct transport and a sub-area of profitability of multi-modal transport against a city of the required rank, located along the main road. The accessibility of these roads for traffic from both sub-areas attracts transport mass and, consequently, investments in technical equipment. Therefore, the routes of a flattened complex typically form the first (against the main road), well-interested road ring. In agricultural areas, the development of investments in peripheral roads



27. Graph of function $K = \psi \cos a + c$ and a hexagonal arrangement. Comparison of the flattening

is underpinned by the effectiveness of circular car transport (Figures 13–16).

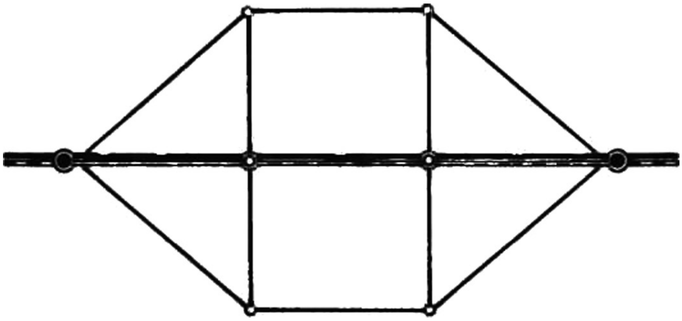
I have called the regular relations between the main and the side roads, represented by the flattening of a complex of side roads along the main road, the law of platination[38].

Expressing platination in measurements of length and the angle is risky. It is more sensible to formulate the identified correlations as an inequality. Therefore, if we adopt a regular hexagon as the starting point, then the deviations of the anisotropic model can be recorded in the following way:

$$\frac{X_a}{Y_a} > \frac{X_h}{Y_h} \qquad 60° \le \varphi_2^x \le 90°, \ \ \varphi_2^\omega < 60°,$$

where X_a, X_h denote the longitudinal axis (the main road) in the anisotropic model and in the regular hexagon, Y_a, Y_h denote the transversal axis in the anisotropic model and in the regular hexagon, $\varphi^{x}{}_{2}$, $\varphi^{\omega}{}_{2}$ denote the deviation of the side roads: the access and direct road (a part of a ring road) against the main road. Figure 28 presents an anisotropic complex worked out from the conditions of a hypothetical area (Figure 24).

As the distance to the main road grows, the impact of the deviating factors lessens and side roads aim at an equilateral triangular or square arrangement. However, because they need to refer to the roads under the influence of the main road, the distortion continues, assuming irreg-



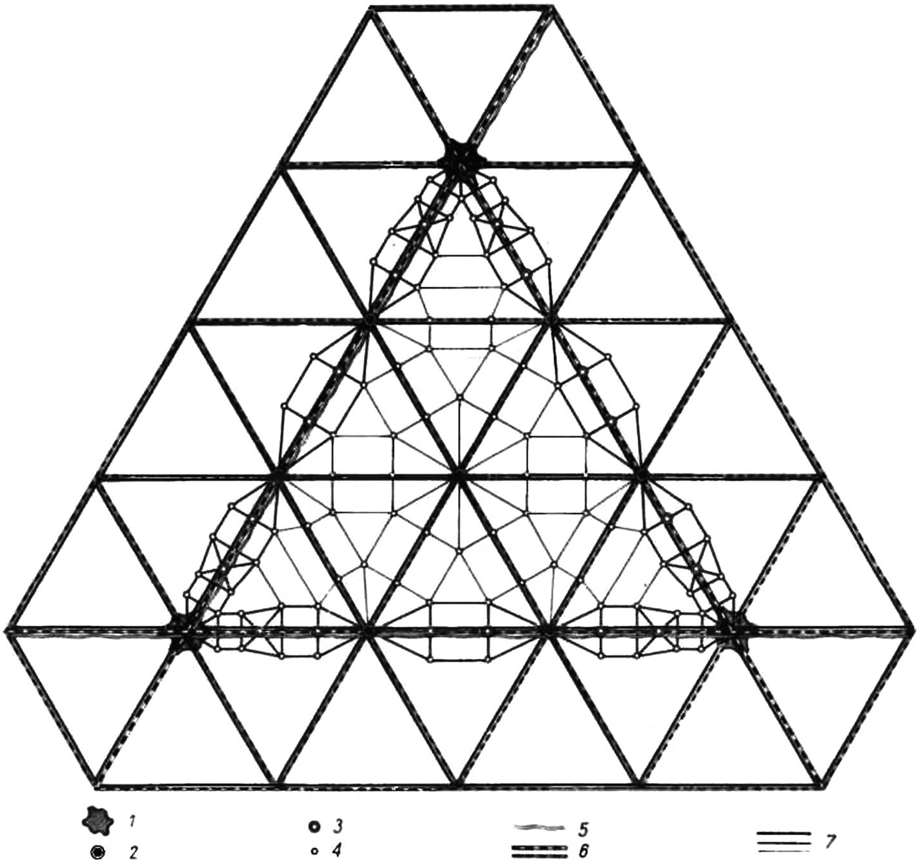
28. A hypothetical anisotropic complex

---

[38] From *Greek platyno*: to flatten, to extend.

ular forms, sometimes opposite. A transversal or diagonal flattening is related to the local concentrations of transported goods.

In the anisotropic model, the length of the side roads is slightly bigger than in the equilateral model (calculating as in Figures 22 and 29, by approx. 2%). Nevertheless, the anisotropic model is more effective because, being better aligned to serving a majority of the traffic along the main roads, it requires smaller costs for transporting goods. This is a major issue. As I have proven before, in the course of development of transport, the share of the costs of roads diminishes while the share of the costs of transporting goods grows.



29. The anisotropic model of a complex of transport networks
1 – class I hubs; 2 – class II hubs; 3 – class III hubs; 4 – class IV hubs; 5 – rivers; 6 – railways; 7 – roads.

4.3. The linear dimensions

A characteristic feature of the anisotropic complex, that differentiates it from the regular hexagonal complex, is the ratio of the number of roads for different types of transport, converging in the hubs (waterways have been disregarded due to excessively irregular occurrences). The ratio of car roads (representing the basic network presented in the anisotropic model) and railways is fixed and amounts to 2, in principle in all the classes of complex nodes (12:6 = 12:6 = 4:2 = 2). Class III hubs are an exception, located in the vicinity of class I hubs where the number of roads is three times bigger. The ratio would be the same in the remaining class III hubs if we introduced to them vicinal roads of diagonal directions. In general, we can write:

$$\frac{S_2^{w\mathrm{I}}}{S_1^{w\mathrm{I}}} \approx \frac{S_2^{w\mathrm{II}}}{S_1^{w\mathrm{II}}} \approx \frac{S_2^{w\mathrm{III}}}{S_1^{w\mathrm{III}}} \approx const.$$

In a regular, hexagonal complex, the ratio of roads and railways amounts to 1 in class I and II hubs which instantly raises reservations (the insufficient differentiation of the properties and functions of various types of transport). Next, in class III hubs, it grows rapidly to 3.

The number of hubs in each class is directly proportional to the size. The bigger the hub the smaller the class while the large classes consist of small hubs.

The ratio between the length of the roads for the different types of transport always has the same sign as the ratio of the number of roads in the hubs but a different, higher value. In the model presented in Figure 29, the roads (the major network) are 4 times longer than the railroads. Introduction of the vicinal and capillary roads would increase the number to 12–16. The actual ratios in the different countries, as well as the conditioning factors, have been largely discussed in literature on geography and transport. Among strictly general factors, of importance are the ratios of different types of transport in the realm of the costs of transport (the roads and transfer) (λ), the traffic capacity of roads (μ), use of the traffic capacity (ν). The ratio between them and the length of the roads can be written in the following way:

$$\frac{S_2}{S_1} = \frac{1}{\lambda\mu\nu},$$

i.e. the ratio of the length of roads (only adjusted to car traffic) to the length of railways is adversely proportionate to the ratio of own costs, road capacity and use of the road.

The density of roads depends on the size of the required and sufficient service area for each road. The smaller the required and sufficient area (and this is related to the extent of development), the denser the road network. This density can be measured, compared and evaluated from the point of view of fulfilling transport needs and making the territory available, by means of Engel's law, supplemented by Uspienski [78] (beside the length of the roads S and the territory A, the law considers the population L and the amount of transported goods $\varPhi$):

$$G_S = \sqrt[3]{\frac{S}{A} \cdot \frac{S}{L} \cdot \frac{S}{\varPhi}} = \frac{S}{\sqrt[3]{AL\varPhi}}.$$

In calculating the distance between the meshes, strictly related to the density of the road network, Böttcher's equation comes in handy [39]:

$$R = \frac{A}{\frac{1}{2}S} = \frac{2A}{S}.$$

It means that the average distribution of the meshes is calculated by dividing the surface in the form of a square by half of the length of the roads and treating one half as horizontal parallels and the other half as vertical parallels.

*

The results of the analysis provide many elements required in a geometric approach to an anisotropic complex of transport networks. However, they do not provide a complete picture. The missing elements have been briefly recognised but cannot be expressed in a discourse way i.e. in theorems.

Figure 29 shows an extreme location of side roads against the main roads, namely a location at an angle of 90° (this is a manifestation of the side roads being majorised by the main roads). As a result, the network of roads has become more regular. The network consists of triangles and rectangles therefore in combines the benefits of the two complexes (the drawbacks have been diminished by applying the right arrangement of roads to the desired arrangement of transport needs).

In the vicinity of large cities, on main and side roads alike, the number of class III hubs is growing. As a result, the side roads are shorter and denser. The shrinking distances between the hubs is accompanied by the diminishing market areas and the movement of the centres towards the large cities.

The radial roads of all the types of transport are more important than peripheral roads. This is most distinct in the vicinity of a large city. The first peripheral road, ignoring the road running around a city's edge (to transfer the traffic in different directions), with incorporated sections of poor technical quality, runs fairly away from that large city (outside the middle of the bisector). The bigger a city, the longer the radius of the area of direct service. The large wedges of surface behind the big cities, without the roads regarded in the model of technical classes, represent recreation areas.

V. The typology of complexes of transport networks

The broader the range of the ratios presented here the less satisfying the single model and the more urgent extension thereof by identifying many types. Typological concepts are different from regular classifying concepts. The nature of *differentia specifica* is that typological concepts operate as a model [57]. A model is compared against real objects in order to ascertain their similarity to it. They can be arranged in an orderly series by means of the appropriate ordering concepts (more frequently used in an intuitive rather than defined way).

There are two types: ideal (unrelated to any object by designation) and empirical (related to at least a single object by designation). The ideal types are not at all free concept fictions. They consist of features dispersed among real objects (highlighted and complex), with learning about empirical reality in mind while the directives, if followed, guarantee their scientific usefulness.

The typological concepts perform three functions: terminology- and classification-related and heuristic. They allow to detect and explain the differences in the occurrence of specific phenomena, the area of interest of idiographic sciences, and to ascertain correctness the mechanism of phenomena which is the role of nomotettic sciences.

When creating the typological contexts, one can use a single or many features of the object in question. When we use only one feature which needs to be gradable, it is the quasi type. On the other hand, when the number of features grows to n (the n-dimensional type). In this case, some of the features can be gradable, other dichotomous or they all can be dichotomous. When the types are identified, two factors in particular need to be taken into account. They include [57]: 1. The point of view from which an object in question is viewed, 2. The type of the assumed causative genetic relations.

In the case of complexes of transport networks, the adopted viewpoint is the spatial structure defined by three features: the hierarchy of roads, the location of roads against each other and the linear dimensions. Among the causative relations, of importance is the dependence of the spatial structure on the functions of different types of transport, historical and geographic conditions. All these relations have already been viewed but only in the most general indications. Further on, less general indications need to be considered, forming characteristic groups. With so many features and relations, the typology of the complexes would need to be extremely sophisticated. The theoretical task at which I am aiming provides a concise approach.

The model of the anisotropic complex combines two arrangements of roads: the triangular and the rectangular (with their optimum in the form of equilateral triangles and squares), each of which can represent the frame for the spatial structure of a different type. They include: 1. the type of complex with a triangular structure and 2. the type of complex with a rectangular structure. The first emerges in the conditions of an organic evolution, in lowlands; the other one typically results from arbitrary shaping in lowlands and mountainous areas alike provided that long valleys are connected by transverse valleys.

To add more details to the two types, they can be further divided into the following sub-types.

1. The odontropic sub-type[39], turned towards the main road. It is created on the coastline, bridging and newly developed areas as well as highly industrialised ones if the main road (e.g. a navigable river) has an exceptionally favourably direction and technical and economic properties. The main road attracts more important centres of the economic life and (or) transit. Simple hierarchy is connected with the biggest diversity of the importance of the side roads and the main road. In the complex of side roads, perpendicular and diagonal access roads prevail while oval roads are sporadic (in large-scale complexes)

---

[39] From Greek ŏdós: a road, and *trópos*: a turn.

or are non-existent. As a result of rarefying the side roads, the ratio of their length and the length of the main road is small.

2. The exotropic sub-type[40], turned towards the outside. Typically, its main hub is a large sea port collecting roads from the rear parts, whose economy is oriented to intense trade exchange with abroad. Over time, a port of this type can extend to a large urban and industrial agglomeration and become, by itself, an important goal of transport from the rear parts. This sub-type is represented by complexes with not very well-developed hierarchy but a large difference in transport and technical equipment of the main and side roads. In the spatial structure, radial roads prevail, concentrated in the main hub. The network of side roads is connected with them in a dendritic arrangement, or is also radial. Typically not fully developed, it contributes to a small ratio of the length of side and main roads.
3. The polycentric sub-type. It emerges in mining and industrial areas with group settlement arrangements and industrial-farming areas with relatively balanced networks of centres. This type is attributed to complexes with a rather complex hierarchy but a moderate difference in the importance of roads. Each centre is a hub of radial, main and side, roads. The arrangement of side roads is complete and relatively most regular. If the centres are spatially compact, in the direction-related arrangement and the hierarchy, direct roads prevail. Complete emergence of a network of side roads increases the ratio of the network's length and the length of main roads.
4. The monocentric sub-type. It is formed in the conditions of strong of political-administrative or economic-spatial centralisation. The sub-type enjoys the most complex hierarchy and significant diversity of the importance of roads. Majorisation of social and economic life by a single centre equals domination of radial roads. These roads assume an arrangement of side roads and deviate it; as a result, the external sections of the oval roads form new radial roads. There is a type of polarization of the arrangement of side roads because, at the same time, the access nature of some of the side roads is highlighted. The new radial lines elongate the network of side roads. As a result, in

---

[40] From Greek é*ksō*: to the outside, and *trópos*: a turn.

the monocentric sub-type, the number of roads in a hub as well as the ratio of side and main roads enjoy the highest values.

In the subsequent sub-types, the hierarchical system is further complicated and the ratio between the length of the side road and the main roads grows. The spatial arrangement of the side roads against the main roads starts with single access roads, perpendicular and diagonal, through dendrite-like connections, to a complete arrangement which is ultimately deformed by the emergence of new direct roads from the side roads, over time transformed into the main roads; this growth is in line with the parallelisation theorem.

The lower-rank typological concepts include classes of complexes, types of complexes and varieties of complexes. They can be obtained by subsequent gradation of development and diversification of a complex' spatial conditions. The latter consists not only in highlighting the typological units of the geographic environment (presented in Chapter III § 4) and spatial development but also in regarding the spatial scale of complexes. As a result of gradating this feature of complexes, we can identify elementary complexes, regional complexes: micro-, mezzo- and macro complexes, provincial, national complexes, zonal and continental complexes.

In the typology of complexes which takes into consideration the historical conditions of their development, convergence complexes occupy a special place. They are defined for the complexes of a specific type (sub-type, class etc.) which have the features of another type, acquired in the course of adjusting to the changed environmental conditions, namely economic. Introduction of diagonal roads to a rectangular arrangement is particularly frequent, making it more similar to a triangular arrangement. This happens, among other circumstances, when a transfer occurs from belt-like to superficial area development; it is accompanied by growth of the centres i.e. also the centripetal transport arrangement.

The identification of typological concepts of actual complexes is limited to individuals who are similar enough to the models. Even if the typology is very extended a certain number of actual complexes is not similar enough (while the similarity is hard to indicate in a clear-cut way). These complexes are referred to as endemic.

VI. Verification

This chapter does not reflect an ambition of to verify the anisotropic model or the identified types and sub-types in an exhaustive way. It will prove its worth if I prove through partial verification that, despite various disturbances, the general features of a theoretical complex will come true and the related further work is advisable.

The applied verification procedure includes two stages: geographic verification and statistical verification.

Geographic verification consists in defining transport zones within which complexes are similar to this or other type or sub-type. Carried out in a narrower regional division, the verification wo0uld in fact be identification of real complexes that would have to be further investigated by means of cartographic or statistical methods against the theoretical model with respect to compliance. Identification of roads in complexes would require data about every road. Collecting these data is (practically) impossible nor is it necessary. We can assume that maps, largely generalised, do not contain incidental elements of transport networks (eliminate them)[41] and these were the maps I used in the light of unavailable detailed maps for a majority of countries and incomparability of the available maps (various criteria of classifying roads as marked and disregarded). Atlases tend to be relatively most homogenous with regard to transport networks in the specific countries. Unfortunately, there are no updated and sufficiently detailed road atlases of the world (for all the three types of transport). So I had to resort to general geo-

---

[41] Contemporary complexes include often relics of the former complexes. Their original functions have stopped; they are just dead remains or perform a different function, e.g. the former navigable canals are used as tourist trails.

graphic atlases and chose, for obvious reasons, the biggest and the latest like The Times Atlas of the World [101] and *Atlas Mira* [2]. General transport maps of the continents proved also helpful, carefully developed by G. Kohler, G. Sendler and Ch. Clauss [50], coupled with national road atlases.

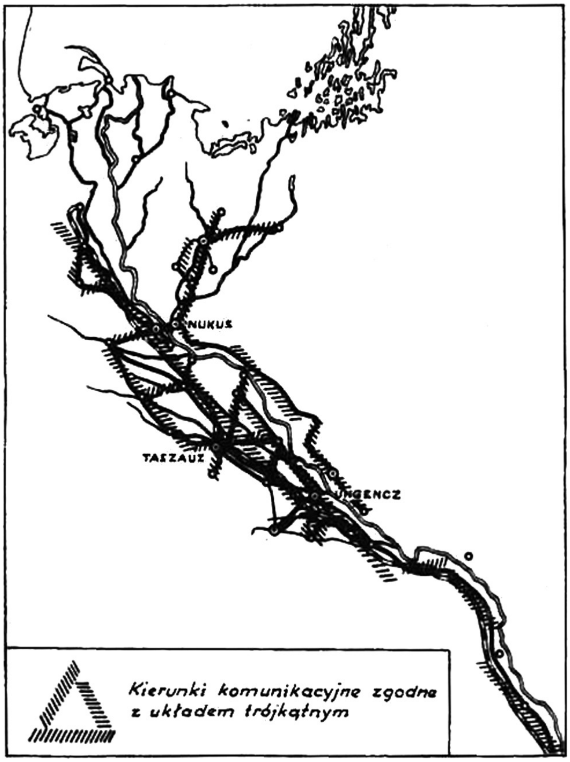
I have already stated that in their most typical form, complexes of transport networks with a triangular structure emerged in Europe while complexes with a rectangular structure emerged in North America (the road network is amazingly regular). The division of the remaining continents is less distinct. In general, the triangular arrangement is characteristic of continental Asia and the rectangular arrangement for the Asian isles; it is conditioned, among other things, by the isles' vertical and horizontal configuration. In Africa and Australia, two types have developed with the prevailing triangular type; the same holds true for South America. On the latter three continents, the emergence of complexes of transport networks has mostly only started; it may develop towards the rectangular or triangular solutions, under the influence of the consolidating national economy and its spatial structure.

Where are the subtypes of complexes most represented?

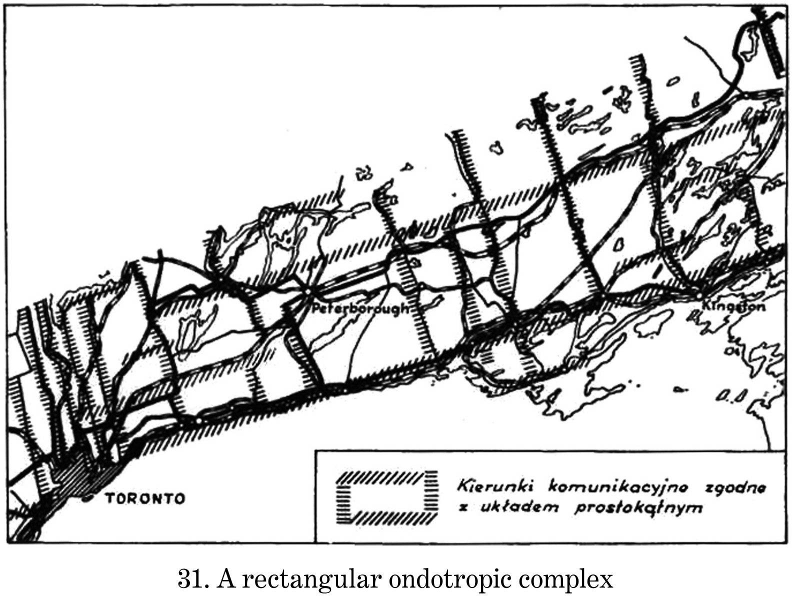
1. The odontropic sub-type

A. The class of complexes with the triangular arrangement (amorphous in originated complexes): Central Asia, the river valleys of South-Eastern Asia and some European rivers, the valley of the Nile. Examples: the Amy Darya system, the Orenburg-Tashkent system, the Karaganda system, the Turkestan-Siberian system, the systems built along the Trans-Siberian Railway and the South-Siberian Railway, the so-called Silk Road system, the so-called sultan road system, the Yangtze system, the Ganges system, the Rheine system, the Rhône system, the Cairo-Aswan system (Figure 30).

B. The class of complexes with the rectangular arrangement: the coastline and the Great Plains of North America, the western coast of South America, Asian islands, the eastern coast of Italian Peninsula and Scandinavian Peninsula: the Mobile-Tallahassee complex, the Jacksonville-Miami complex, the Saint Lawrence Seaway, the Toronto-Kingston complex, the Winnipeg-Regina-Lethbridge complex, the Kansas City-Denver complex, the Antofagasta-



30. An triangular odontropic system



31. A rectangular ondotropic complex

Coquimbo complex, the Santiago-Concepción complex, the Djakarta-Semarang-Surabaya complex, the Surabaya-Djokjakarta complex, the Djokjakarta-Sukabumi complex, the Sendai-Hacinoe complex, the Niigata-Akita complex, the Kumamoto-Kagoshima complex, the Rimini-Ancona-Pescara-Termoli complex, the Taranto-Corigliano complex, the Crotone-Reggio di Calabria complex, the Umea-Boden-Haparanda complex (Figure 31).

2. The exotropic sub-type

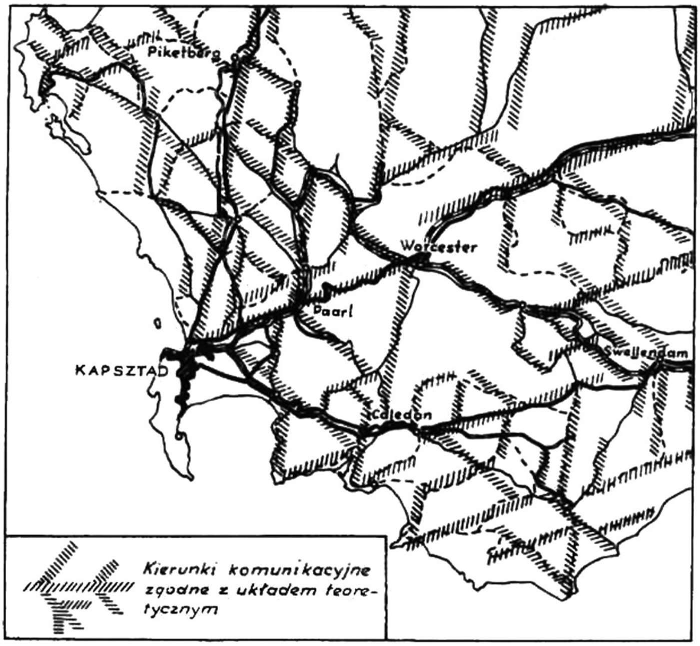
A. The class of complexes with the triangular (or radial) arrangement: zones of services rendered to Europe's sea ports (in singular cases, the complexes in the zones of the biggest ports may assume quasi monocentric shapes), south and north Africa, south-eastern Asia, south-eastern Australia, the countries of the Rio de la Plata Basin and Brazil. Examples: the London complex, the Hamburg complex, the Szczecin complex, the Gdansk complex, the Leningrad complex, the Cape Town complex, the Port Elizabeth complex, the Tunis complex, the Bombay complex, the Calcutta complex, the Canton complex, the Shanghai complex, the Melbourne complex, the Sydney complex, the Buenos Aires complex, the Montevideo complex, the Sao Paulo complex, the Rio de Janeiro complex (Figure 32).

B. The class of complexes with the rectangular arrangement: zones of services rendered to sea ports of North America (especially medium-sized and small ones)[42], West Australia, Upper and Lower Guinea (complexes originated with a single main direction). Examples: the New Orleans complex, the Corpus Christi complex, the Pensacola complex, the Tampa complex, the Portland complex, the Quebec complex, the Seattle complex, the Perth complex, the Conakry-Kankan complex, the Matadi-Leopoldville complex, the Luanda-Malanje complex (Figure 33).

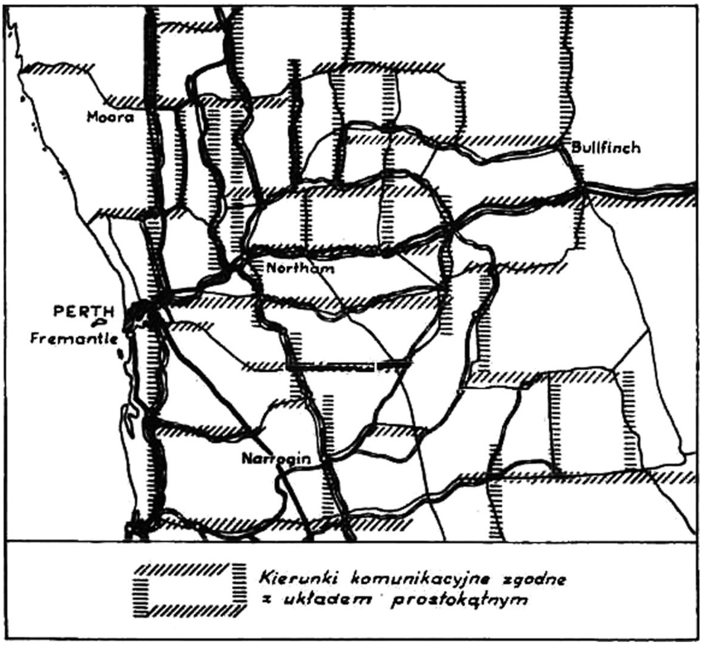
3. The polycentric sub-type

---

[42] In the zones of services rendered to metropolitan cities, the rectangular, exotropic and monocentric complexes transfer into convergent complexes. This feature differentiates rectangular exotropic and monocentric complexes from triangular exotropic and monocentric complexes which tend to assume the most characteristic forms on a large scale.

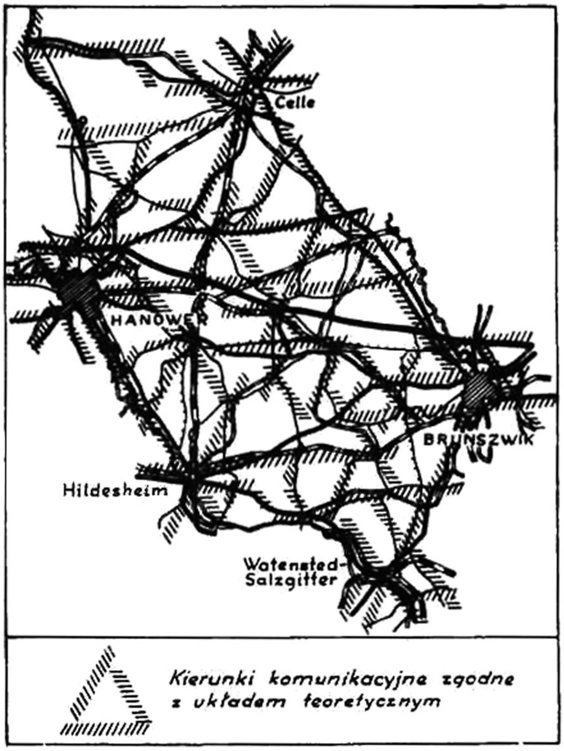


32. Exotropic triangular complex



33. Exotropic rectangular complex

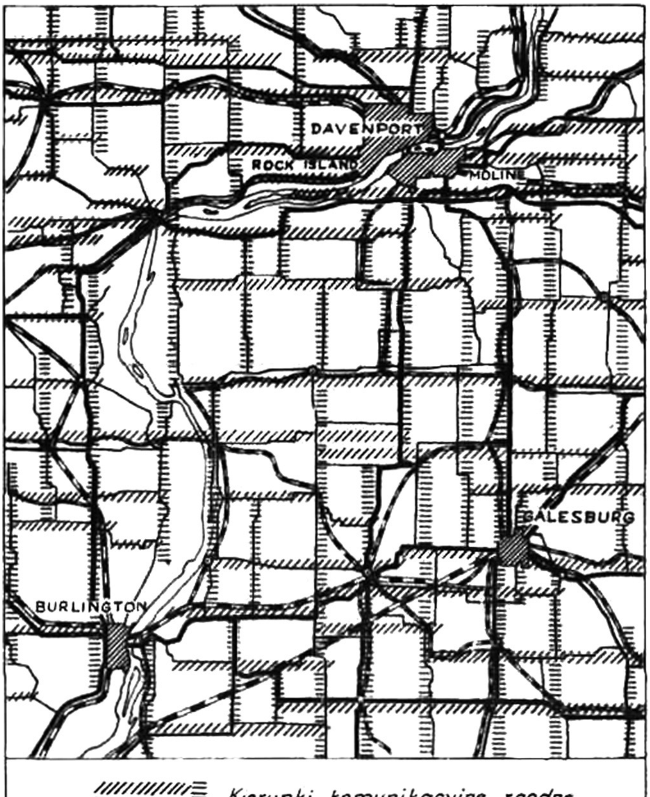
A. A class of subtypes with a triangular arrangement: European countries strongly fragmented at the time of feudalism (especially Germany), consolidated in a network of roads and interchanges referred to in the subsequent development of railways and industry; Europe's mining and industrial centres; sporadically in south-eastern Asia. Examples: the Leeds-Bradford-Halifax-Huddersfield complex, the Dortmund-Essen-Duisburg-Düsseldorf-Wuppertal complex, the Braunschweig-Salzgitter-Hildesheim-Hannover-Celle complex, the Lüneburg-Uelzen-Dannenberg complex, the Elsterwerda-Grossenhain-Riesa complex, the Bydgoszcz-Torun complex, the Upper Silesian complex, the Donetsk complex, the Mukden-Changchun-Harbin complex (Figure 34).



34. Polycentric triangular complex

B. A class of rectangular complexes: the Great Plains, the Great Lakes and the eastern mining and industrial centres of North America. Examples:

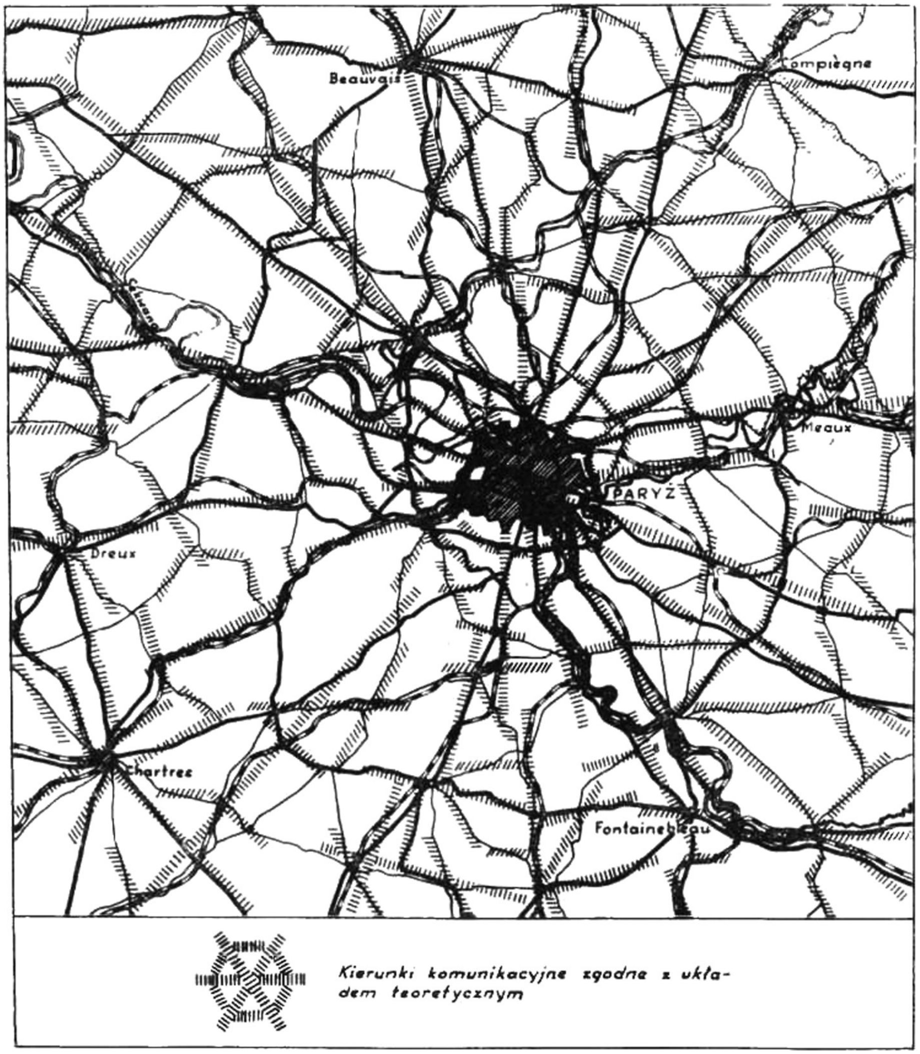
the Davenport-Rock Island-Galesburg-Burlington complex, the Cedar Rapids-Iowa City-Davenport-Clinton complex, the Rochester-Austin-Albert Lea-Owatonna complex, the Fort Scott-Pittsburgh-Parsons-Chanute complex, the Ardmore- Marietta-Gainesville-Sherman-Durant complex, the Grand Rapids-Lansing-Jackson-Battle Creek-Kalamazoo complex, the Kitchener-Guelph-Galt-Waterloo complex, the Pittsburgh-Cleveland-Akron-Canton-Youngstown complex, the Harrisburg-Allentown- Scranton-Sunbury complex (Figure 35).



35. Polycentric rectangular complex

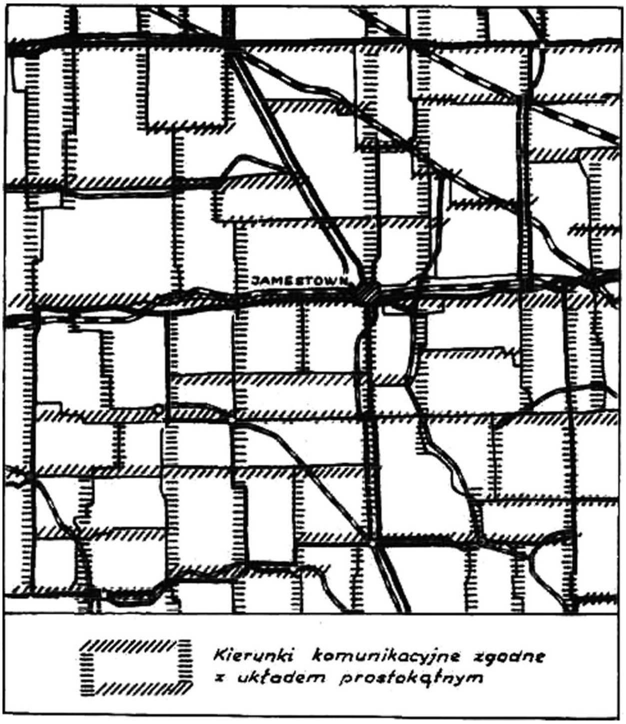
4. The monocentric sub-type

A. A class of triangular complexes: zones of services rendered to the metropolitan cities in Europe and south-eastern Asia (beside sea ports). Examples: the Moscow complex, the Warsaw complex, the Berlin complex, the Paris complex, the Budapest complex, the Vienna complex, the Milan complex, the Madrid complex, the Delhi complex, the Heydarabad complex, the Wuhan complex, the Czongqing complex (Figure 36).



36. Monocentric triangular complex

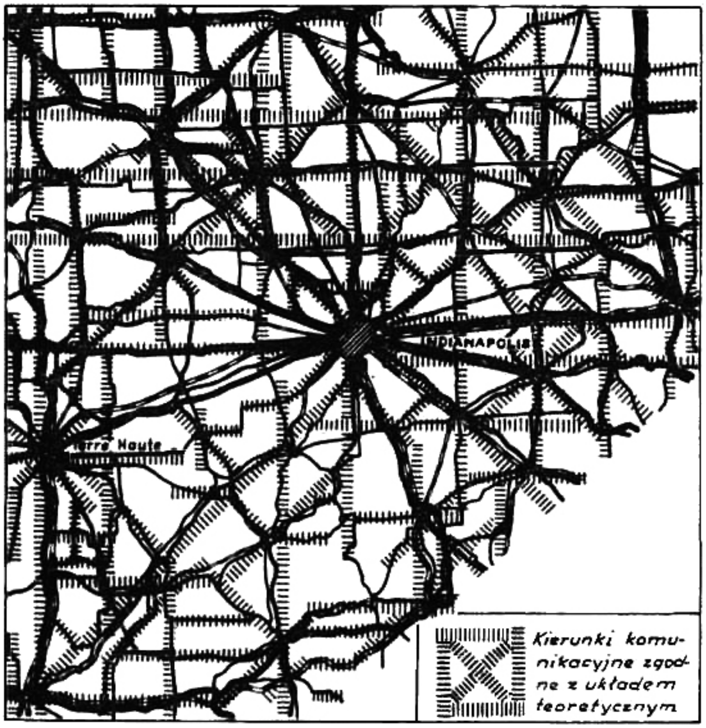
B. A class of rectangular complexes: the interior of North America, especially the zones of medium-sized and small cities. Examples: the Calgary complex, the Edmonton complex, the Flint complex, the Wausau complex, the Fargo-Moorhead complex, the Jamestown complex, the Aberdeen complex, the Watertown complex, the Des Moines complex, the Omaha complex, the Topeka complex, the Wichita complex, the Denver complex, the Pueblo complex, the Oklahoma City complex (Figure 37).



37. Monocentric rectangular complex

The convergent variety of rectangular, monocentric and exotropic complexes: the zones of services rendered to metropolitan cities in North America (inland monocentric convergence complexes, exotropic convergence complexes on the coast). Examples: the New York complex, the Philadelphia complex, the Cincinnati complex, the Indianapolis complex, the Chicago complex, the Detroit complex, the Minneapolis-Saint

Paul complex, the Kansas City complex, the Saint Louis complex, the Winnipeg complex (Figure 38). The aim of statistical verification is to ascertain if the theoretically expected relations in fact take place and to what extent (in comparison with the model). The verification is limited to invariant features i.e. ones that do not change with typological transformations and are maintained in historical complexes; in other words, it is limited to features typical of an anisotropic model. Simple statistical methods will suffice.



38. Convergent variety of monocentric rectangular complex

The occurrence of the first feature i.e. the advantage of the main road over the sum of the relevant side roads, expressed in the number of transports, can be verified if you have at your disposal the results of detailed traffic measurements. In practice, the verification can only be partial. The Poznań region was selected as an experimental transport region with its 3,219 km of railways and 31,774 km of roads. The resulting transport network was divided into 36 complexes. The coefficient of concordance, established in the course of comparing transports, amounted to: for complexes consisting of a railway and roads 1.0 (complete concordance); for complexes consisting of a main road and railways 0.92.

In the verification of the second feature, platination, an agreement was applied that instead of the location of roads against railways, we would check the location of side roads against the main roads. This is acceptable because in regions with well-established transport networks, railways also run in the same directions as main roads. While the routes are different their differentiation would involve complications; bearing in mind the advancement of the complex theory, consideration thereof would not bring about useful results. All the important transport regions in the world with existing networks of roads were examined with respect to platination. First, road junctions that could form complexes were examined; there were 890 of them. Next, we checked which of them were compliant with a hypothetical anisotropic complex (Figure 28). Complexes which had at least two out of the three properties of a hypothetical complex were adopted as compliant (if two properties were allocated to a complex in half, they were considered a single value). The coefficient of concordance amounted to 0.84. If was still quite high even when the calculation encompassed only the complexes with all the three properties; then the coefficient amounted to 0.70.

In order to verify the third feature – the constancy of the quantitative ratio between roads and railways in transport hubs – all the large hubs in more important transport regions of the world (367 hubs) were examined. To obtain comparable ratios, only one atlas was used, namely The Times Atlas of the World. In the course of random identification of roads and railways in the hubs an observation was made that on large-scale maps, railways were typically marked in their entirety while only 3/4 of roads were taken note of. Therefore, the identified number of roads was extended by 1/4. The data on the quantitative structure of the hubs allowed us to calculate the dispersion measures (characterising the structure better than the coefficient of concordance), together with the most important measure, the standard deviation. It was calculated according to the following formula:

$$\sigma_x = \sqrt{\frac{\sum(x - \overline{x})^2}{n}}$$

(where x indicates the individual value of the quantitative ratio between roads and railways, $\overline{x}$ indicates the ratio's value in the anisotropic model, indicates the number of examined hubs), the deviation amounts to barely 0.77. The distribution of the ratio's values is quite symmetric and its drawing is reminiscent of Gauss distribiution.

Bibliography

Annual Bulletin of Transport Statistics 1955. United Nations. Geneve 1956.

Atlas Mira. Moscow 1955.

Barciński F., Wstęp do nauki geografii gospodarczej. Poznań 1935.

Barciński F., Człowiek zmienia oblicze Ziemi. Warsaw 1953.

Batalow A.L., Transport w sowremiennoj Indii. Moscow 1961.

Berezowski S., Geografia transportu. Warsaw 1954.

Berezowski S., Przegląd dorobku francuskiej geografii transportu. In: „Przegląd Zagranicznej Literatury Geograficznej” 1956, ed. 2, pp. 15–33.

Berezowski S., Podstawy metodologii geografii transportu. Warsaw 1961.

Biełousow I.I., Komarow A.W. (eds.), Problema wzaimodiejstwija razlicznych widow transporta. Moscow 1961.

Bissaga T., Geografia kolejowa Polski. Warszawa 1938.

Blum O., Verkehrsgeographie. Berlin 1936.

Böhm K.P., Die Integrierung und Koordinierung der Güterverkehrsmittel in Grossbritannien. 1959.

Bujak F., Z odległej i bliskiej przeszłości. Lvov–Warsaw–Krakow 1924.

Capot-Rey R., Geographie de la circulation sur les continents. Paris 1946.

Chaczaturow T. S., Ekonomika transporta. Moscow 1959.

Chaczaturow T.S. (ed.), Razwitije proizwoditielnych sił Wostocznoj Sibiri. Transport. Moscow 1960.

Chanukow E.D., Transport i razmieszczenije proizwodstwa. Moscow 1956.

Chojnicki Z., Ocena dorobku polskiej geografii transportu. „Przegląd Geograficzny”, T. 29: 1957, ed. 2, pp. 317–338.

Chojnicki Z., Analiza przepływów towarowych w Polsce w układzie międzywojewódzkim. Warsaw 1961.

Christaller W., Die zentralen Orte in Süddeutschland. Jena 1933.

Cleef E., Trade Centres and Trade Routes. New York 1937.

Comey A. C., Regional Planning Theory. Cambridge, Mass. 1923.

Dagget S., Principles of Inland Transportation. New York 1955.

Dantzig G.B., Application of the Simplex Method to a Transportation Problem. W: Activity Analysis of Production and Allocation. Ed. by T. C. Koopmans. New York–London 1951.

Domański R., Lokalizacja przemysłu wobec zmian w systemie transportowym. „Ruch Prawniczy i Ekonomiczny", vol. 21: 1959, ed. 1, pp. 221–235.

Domański R., Metody analizy układu sieci drogowej. PAN Komitet Przestrzennego Zagospodarowania Kraju. Biuletyn, ed. 3 (5) 1961, pp. 89–109.

Dziewoński K., Zasady przestrzennego kształtowania inwestycji podstawowych. Warszawa 1948.

Dziewoński K., Geografia osadnictwa i zaludnienia. Dorobek, podstawy teoretyczne i problemy badawcze. „Przegląd Geograficzny", vol. 28: 1956, ed. 4, pp. 721–764.

Dziewoński K., Elementy teorii regionu ekonomicznego. „Przegląd Geograficzny" vol. 33: 1961, ed. 4, pp. 593–613.

„The Eastern Economists". New Delhi. Annual Number 1956.

Economic Survey of Europe in 1956. United Nations. Geneva 1957.

Economics of Highway Trailers on Flat Cars Service. „Bulletin of American Railway Engineering Association", vol. 57: 1955, no. 525.

Friedrich E., Schmidt W., Geographie des Welthandels und des Weltverkehrs. Jena 1930.

Friedrich P., Die Variationsrechnung als Planungsverfahren der Stadt- und Landesplanung. Bremen-Horn 1956.

Frisch K., Życie pszczół. Biblioteka Wiedzy, vol. 16, Warsaw.

Garrison W. L., The Benefits of Rural Roads to Rural Property. Seattle 1956.

Garrison W.L., Marble D.F., Analysis of Highway Networks: A Linear Programming Formulation. Washington: Highway Research Board, Proceedings, 37: 1–14. 1958.

Gierałtowski Cz., O zastosowanie bodźców ekonomicznych w walce o racjonalizację przewozów. „Transport”, 1956, no. 7.

Hasser K., Allgemeine Verkehrsgeographie. Bd. 1–2, Berlin und Leipzig 1931.

Haufe H., Die geographische Struktur des deutschen Eisenbahnverkehrs. Langensalza–Berlin–Leipzig 1931.

Hettner A., Allgemeine Geographie des Menschen. Bd. 3. Verkehrsgeographie. Bearbeitet von H. Schmitthenner. Stuttgart 1952.

Hoover E.M., The Location of Economic Activity. New York–Toronto–London 1948.

Isard W., Location and Space-Economy. New York 1956.

Isard W., Methods of Regional Analysis: an Introduction to Regional Science. New York–London 1960.

John J. St., The Railways of Britain. London 1954.

Kantorowicz Ł.W., O pieremieszczenii mass. DAN SSSR, vol. 37: 1942, no. 7–8, pp. 227–229.

Kantorowicz Ł.W., Gawurin M.K. [Primienienije matiematiczeskich mietodow w woprosach analiza gruzopotokow. W: Problemy powyszenija effektiwnosti raboty transporta. Moscow 1949.

Kehr C., A National Plan. New York 1926.

Kohl J.G., Der Verkehr und die Ansiedelungen der Menschen in ihrer Abhängigkeit von der Gestaltung der Erdoberfläche. Leipzig 1850.

Köhler G., Sendler G., Clauss C., Verkehrsgeographische Übersichten der Kontinente. „Petermanns Geographische Mitteilungen”, vol. 100: 1956, vol. 101: 1957, vol. 102: 1958.

Koopmans T.C., Optimum Utilization of the Transportation Systems. „Econometrica”, vol. 17: 1940, Supplement, pp. 136–146.

Kuhn H.W., On Certain Convex Polyhedra. „Bulletin of the American Mathematical Society”, Vol. 61: 1955, no. 6, pp. 557–558.

Lange O., Ekonomia polityczna. T. I. Zagadnienia ogólne. Warsaw 1959.

Lartilleux H., Geographie des chemins de fer français. Geographie universelle de transport. Paris 1947–1950.

Launhardt W., Die Bestimmung des zweckmässigsten Standortes einer gewerblichen Anlage. „Zeitschrift des Vereins deutscher Ingenieure”, Bd. 26: 1882, no. 3, pp. 106–115.

Launhardt W., Theorie des Trassierens. Hannover 1887.

Lazari-Pawlowska L, O pojęciu typologicznym w humanistyce. „Studia Filozoficzne", 1958, ed. 4(7), pp. 30–53.

Leszczycki S., Transport autobusowa w województwie krakowskim. Wiad. Stow. Czł. Pol. Kongr. Drogowych, vol. 4: 1930, no. 34.

Leszczycki S., Nowsze kierunki i prądy w geografii. „Przegląd Geograficzny", vol. 30: 1958, ed. 4, pp. 543–571.

Leszczycki S., Geografia stosowana czy zastosowanie badań geograficznych dla celów praktycznych. „Przegląd Geograficzny", vol. 34: 1962, ed. 1, pp. 3–23.

Lewis R.G., Handbook of American Railroads. New York 1956.

Linden W., Verkehr und Raumordnung. W: Raum und Verkehr I. Forschungs-und Sitzungsberichte der Akademie für Raumforschung und Landesplanung. Bremen-Horn 1956.

Locklin D. Ph., Economics of Transportation Homeowod, Illinois 1954.

Lösch A., Gospodarka przestrzenna. Przekład z niemieckiego. Warsaw 1961.

Luce R.D., Raiffa H., Games and Decisions. New York 1957.

Łopuszyński M., Podstawy rozwoju sieci komunikacyjnej w Polsce. Warsaw 1939.

Łopuszyński M., Podstawowe zagadnienie polityki komunikacyjnej. Warsaw 1947.

Łukaszewicz J., Steinhaus H., O wyznaczaniu środka miedzi sieci telefonicznej. „Zastosowania Matematyki", vol. 1: 1954, ed. 4, pp. 299–307.

Łukjanow N.A., Srawnitielnaja ekonomiczost' ispolzowanija razlicznych widów transporta. In: Woprosy ekonomiki żeleznodorożnogo transporta. Transżeldorizdat 1948.

Maunier R., La distribution géographique des industries. In: „Revue Internationale de Sociologie", 16 Annee, 1908.

Mc Lean W.H., Regional and Town Planning in Principle and Practice. London 1930.

Meyer J.R.,Peck M.J., Stenason J., Zwick Ch., The Economics of Compétition in the Transportation Industries. Cambridge, Mass. 1959.

Mio decki W., Koordynacja transportu kolejowego i samochodowego. „Transport", 1954, no. 9.

Morgenthaler, Wollert, Gutachten über die kostenmässigen und tariflichen Grundlagen des Leistungswettbewerbes zwischen der Bundesbahn und dem Güterfernverkehr mit Lastkraftwagen. Düsseldorf 1952.

Müller P., Schiene-Strasse und Schiene-Schiffahrt. Düsseldorf.

Mumford L., Technics and Civilization. New York 1940.

Neft D., Some Aspects of Rail Commuting: New York, London and Paris. „Geographical Review”, Vol. 49: 1959, no. 2.

Nikolski I.W., Gieografija transporta SSSR. Moscow 1960.

Nye J.F., Physical Properties of Crystals. Oxford 1957.

Otremba E., Allgemeine Geographie des Welthandels und des Weltverkehrs, Stuttgart 1957.

The Oxford Atlas. London 1958.

Palander T., Beiträge zur Standortstheorie. Uppsala 1935.

Palatas Z., A korszerü közlekedesi munkamegosztas alapjai. Budapest 1959.

Picard R. M., Bilan d’exploitation simplifiée et modernisée d’une petite ligne. „Revue Générale des Chemins de Fer”, Août 1956.

Pirath C., Die Grundlagen der Verkehrswirtschaft. Berlin–Göttingen–Heidelberg 1949.

Pirath C., Das Selbstkostenproblem Schiene–Strasse. In: „Schiene und Strasse” 1953, Dortmund.

Pirath C., Die Verkehrsteilung Schiene–Strasse in landwirtschaftlichen Gebieten und ihre volkswirtschaftliche Bedeutung. Berlin–Göttingen–Heidelberg 1954.

Predöhl A., Das Standortproblem in der Wirtschaftstheorie. „Weltwirtschaftliches Archiv”, Vol. 21: 1925, pp. 294–321.

Remy K., Die Eisenbahnen im Rahmen des afrikanischen Gesamtverkehrs. In: Afrika. Handbuch der praktischen Kolonialwissenchaften. Bd. XV, Teil 1. Berlin 1943.

Richthofen F., Vorlesungen über allgemeine Siedlungs- und Verkehrsgeographie. Berlin 1908.

Rittershausen H., Bundesbahn im Wettbewerb, die Umstellung eines grössten ehemaligen Monopolunternehmens auf eine neue Zeit. Bad Hersfeld. 1950.

Sapper K., Allgemeine Wirtschaftsund Verkehrsgeographie. Leipzig–Berlin 1925.

Sauszkin J., Wstęp do geografii ekonomicznej. Warsaw 1960.

Sax E., Die Verkehrsmittel in Volks-und Staatswirtschaft. Bd. 1–2, Berlin 1918–1920.

Schmidt P.H., Die geographischen Grundlagen der Verkehrswirtschaft. Jena 1934.

Schwarz I.J., Verkehrskoordinierung. Schiene, Strasse, Luftfahrt, Wasserweg im Wirtschafts-kampf. Wien 1952.

Sierks H.L., Grundriss der sicheren, reichen, ruhigen Stadt. Dresden 1929.

Smirnow A.I., Wybór wniieszniego transporta promyszlennych predprijatij. Trudy Wsiesojuznogo Naucznogo-Issledowatielskogo Instituta Zeleznodorożnogo Transporta. Transżeldorizdat 1959.

Stackeiberg H., Das Brechungsgesetz des Verkehrs. „Jahrbücher für Nationalökonomie und Statistik", Bd. 148: 1938, pp. 680–696.

Thünen J.H., Der isolierte Staat in Beziehung auf Landwirtschaft und Nationalökonomie. Hamburg 1826.

The Times Atlas of the World. Vol. I–V. London 1955–1959.

Tołstoj A.N., Mietody ustranienija nieracjonalnych pierewozok pri planirowanii. „Socjalisticzeskij Transport", 1939, no. 9.

Ullman E.L., Transportation Geography. In: American Geography. Inwentory and Prospect. Syracuse N. Y. 1954.

Wasiutyński Z., O kształtowaniu układów komunikacyjnych. Warszawa 1959.

Watermann H., Die Koordinierung des Eisenbahn- und Strassengüterverkehrs in Frankreich. 1955.

Weber A. Uber den Standort der Industrien. Teil I: Reine Theorie des Standortes. Tübingen 1909.

Wróbel A., Godowski S., Wojtyniak J., Kompleksowe badania przestrzennej struktury przewozów towarowych w Polsce (założenia, cele, metody). PAN Komitet Przestrzennego Zagospodarowania Kraju. Biuletyn, ed. 3(5) 1961, pp. 5–43.

Wrzosek A., Z geografii komunikacyjnej Pomorza. Toruń 1935.

Zwonkow W. W., Wzaimoswjaź otdielnych widów transporta i osnowy organizacji smieszannych pieriewozok. Moscow 1953.

Żeleznyje dorogi mira. Przekład z angielskiego. Moscow 1959.

Subject index

Allocation: in transport it indicates a division of its resources (roads and means of transport) into various areas of transport work. This division may indicate specific roads and means of transport to handle transport of specific loads and passengers or, more interestingly from the point of view of economic geography, specific spatial ranges (close, medium or far range or specific areas, lines and settlements).

Anisotropy: the physical properties of materials can be divided into dependent and independent from the direction. In physics, the former are called isotropic (e.g. the density of crystals), the latter are called anisotropic (e.g. the conductivity of crystals). The concept of anisotropy can be useful in the characteristics of complexes of transport networks. It turns out that their major properties are anisotropic i.e. changing according to the direction. In particular, the properties of the directions of the main roads and the side roads are different. This simple differentiation is of great importance to explaining the spatial arrangement of complexes. It replaces the most simplifying assumptions which are at the basis of regular geometric systems (hexagonal, triangular): the homogeneity of transport, the homogeneity of space and statics. This is because the main and side directions are serviced by various types of transport, are the result of an evolution of transport, they are both the cause and effect of space differentiation. Therefore, the differentiation of the main and side directions reflects the synthesis of the new premises introduced by me which bring geometric charts closer to reality. Mathematical methods have also proved useful in studying the complexes of transport networks; previously the methods were used in research into the anisotropy of crystals

Capillary roads: single roads leading to farms, factories etc. (from Latin *capillaris*).

A centralised network model: a system of equations which determines the shape of a network of roads necessary to connect a specific set of settlements with a specific centre. The shape of the network should fulfil the minimum condition of the transport costs (the costs of roads and traffic).

Complementarity and substitution of transport: complementarity indicates the mutual complementing of roads and means of transport without which transport could not take place. For example, transport by road to a river harbour and further by the river to the port of destination if the river is the only way in the direction of that port; or transporting goods by the river and sections of railway bypassing waterfalls on the river that could not be covered by river ships (navigating the Congo). Substitution indicates the competition in providing the same transports. For example, the competition between river navigation and a railway running in parallel to the river (the Rheine valley).

A convergent complex of transport networks: a complex of a specific type which has features of another type, acquired in the course of adjusting to the changed environmental conditions, mainly economic; for example, a triangular complex, which emerged from a rectangular complex as a result of the development of diagonal directions.

The copper centre model: a system of equations which indicates the location of a telephone switch where its connection with the concentrators where the wires from specific telephones meet; it requires the smallest amount of copper wire. If a copper wire of different lengths is replaced by roads of various shapes, the result will be a transport model.

Dendrite transport system: a system reminiscent of the branching of a tree or shrub.

An endemic complex of transport networks: a complex which occurs only in a specific, very limited area.

An exotropic complex of transport network: an inside-oriented complex; its major hub is typically a large sea port, collecting the back roads, whose economy is based on an intense trade exchange with abroad.

Functions of typological concepts: typological concepts contribute to enhancing concept-related clarity and sharpness, they are used to order and classify concepts and, by comparing specific phenomena and types, make it possible to reveal facts which require explanation or previously unnoticed regularities

Identification: here, the first of two issues of ascertaining concordance or lack of concordance between a scientific theorem and reality. It consists in establishing which components of reality are the equivalents of a scientific theorem. The other issue is establishing the degree of concordance.

The law of platination: regular relations between the main and side roads, expressed in the flattening of the system of side roads along the main road.

The law of refraction of transport: a very general theorem according to which multi-modal transport (with reloading, here – by means of two types of transport) is least expensive when it refracts so the sines of the angles of incidence and refraction of roads are inversely related to the unit costs of both transports; an analogy with the law of refraction.

An odotropic complex of transport networks: a complex turned towards the main road; this road is the axis of the complex with a system of side roads subordinated to it.

The parallelisation theorem: a theorem about the development of the mutual location of roads for different means of transport that says that in their location against the roads of the old means of transport, the roads of the new means of transport initially aim at the perpendicular location and later, through intermediate stages, to a parallel location.

Reorientation of the market: a change in the market area: its location, shape and size against transport roads, following upgrading thereof.

A travelling salesman model: a system of equations indicating the routes between the specific settlements in an area so that the total length of the routes is the smallest. The establishment of this system of routes is a characteristic task faced by a travelling salesman starting his business trip.

Vicinal roads: roads of local importance, forming a network of connections.

Summary

This paper is of theoretical character. It deals with the following three subject matters: the morphology, the typology and the verification of complexes of transport networks. The term: complexes of transport networks is meant to represent a cohesive and harmonized set of tracks of diverse types of transport (these types may be supplements or substitutes of each other), operating towards a common purpose (servicing transport within a defined area or in a defined direction), with a minimum total cost involved.

I presuppose that the formation of such complexes is an orderly process, fit to be investigated in order to recognize its regularities. Of particular significance I consider the interrelation recurring in their hierarchy, in their mutual location and in the number of tracks occurring in these complexes.

The models of transport networks hitherto known are of limited theoretical and practical value, since they were brought into existence due to excessively simplified premises (uniformity of tracks of transport, uniform two-dimensional space, stable conditions). For this reason I have introduced more complicated premises such as: differentiation in the properties of the respective types of transport, differentiation in economic space, and the factor of evolution. These presuppositions led to an anisotropic model of a complex, and I define its premises, its fundamental properties and some fraction solutions.

The point of issue are simple, yet steadily repeated structural relations of transport. These comprise the ratio of safflux to transport proper by transfer transport, and the ratio of transfer transport to direct transport. From these two ratios I deduct, using the law of transport refraction, the elementary complexes of tracks of transport which, ru-

dimentarily, contain the elements of a spatial structure of complexes of transport networks.

The cost of goods transfer ostensibly creates a new boundary to the means of transport, where the directions of lateral tracks towards main tracks are broken supplementarily, deviating into a direction perpendicular to the main tracks.

The factor of evolution sheds light on the formation of parallel tracks in elementary complexes, the permanence of a spatial structure of such complexes in spite of innovations in the forms of transport and, partly also, on "platination", i.e. the flattening of the pattern of side roads with regard to the main road, combined with a widening of this pattern.

Today the effect of the geographic environment on the theoretical models of transport can be considered by introducing into the respective formulae suitably chosen parameters which indicate the obstacles posed to transport by the environment. Here the concept of the "virtual distance" may be utilized. A far-reaching generalization of geometric models seems to be feasible owing to topological methods. Even in areas least differentiated as regards their physical geography, complexes of transport networks show anisotropic features, i.e. features dependent on directions. This fact must be attributed to the requirements of transport itself, as well as to the anisotropy of economic space. The main role is played here by large-scale advantages, either as to production or transport.

The anisotropy of complexes of transport networks appears, inter alia, in the differentiation of the properties of main roads and side roads. The main roads lead to reorientation of market areas. I suggest, as mathematical approximation of changes in the shape of the market area, adoption of the function $K = \psi \cos \alpha + c$, where K represents the shape of the area, ψ = the coefficient of proportionality, and c = the value of a constant independent of the direction. The reorientation of the market areas is connected with deformation of the pattern of side roads with regard to the main road. The deviation of a single road may be defined by the law of transport refraction, with simultaneuos additional differentiation of the cost of transport in both the main and the side direction. In order to determine the deformation of a specified sector of the network of roads, the tensor calculus is applied.

The fundamental elements of an anisotropic model have been established; however, not all of them can thus far be expressed discursively, i.e. in theories. The relative importance (hierarchy) of roads in a complex may be expressed by the inequality:

$$\Phi_1^s > \sum_{j=1}^{n} \Phi_{2\,j}^{s}.$$

$\Phi^s{}_1$ = the stream of transport on the main road, and $\Phi^s{}_2$ = the stream of transport on the side roads. The location of the side roads with regard to the main road is subject to the law of platination. Thus this location differs from that of a regular hexagonal model, the most efficient model in conditions of uniform transport, and of geographic and economic space. These deviations may be expressed as follows:

$$\frac{X_a}{Y_a} > \frac{X_h}{Y_h} \qquad 60^\circ \leq \varphi_2^x \leq 90^\circ,\ \ \varphi_2^\omega < 60^\circ,$$

where X_a, X_h denote the longitudinal axis (the main road) of the anisotropic and the regular hexagonal model, Y_a, Y_h – the transversal axis of the respective models, and $\varphi^x{}_2$, $\varphi^\omega{}_2$ the inclination of the side roads: the feeder road and the direct road (part of the peripheral trend) as regards the main road. A characteristic feature of the anisotropic model, differentiating it from the regular hexagonal model, is the proportion of the number of roads of various types of transport converging in the hubs. The ratio between roads and railways is constant in practically all classes of hubs, and equals 2. Thus this ratio may be expressed as follows:

$$\frac{S_2^{w\mathrm{I}}}{S_1^{w\mathrm{I}}} \approx \frac{S_2^{w\mathrm{II}}}{S_1^{w\mathrm{II}}} \approx \frac{S_2^{w\mathrm{III}}}{S_1^{w\mathrm{III}}} \approx const,$$

where $S^w{}_1$ defines the number of main roads meeting in the hub, and $S^w{}_2$ – the number of side roads in this hub; I, II, III denote the respective class of hubs. A geometric illustration of this anisotropic model is shown in Figure 29.

However, the wider the scope of the interrelations to be taken into consideration, the less satisfactory is one single model and the more imperative is its evolution by distinguishing many types. The anisotropic model comprises two patterns of roads: the triangular and the rectangular pattern (attaining their optimum values in the shapes of equilateral triangles and squares); each of these may again become the pattern of the spatial structure of a different type. We thus may distinguish: 1) a triangular type of complex, 2) a rectangular type of complex. Within each of these types, the following four sub-types may be distinguished: the odotropic, the exotropic, the polycentric and the monocentric sub-type. In the successive sub-types, there appear complications in the hierarchic pattern and an increase in the ratio of the lengths of lateral roads to the lengths of the main roads. The spatial pattern of side roads with regard to the main road develops from single feeder roads of perpendicular or oblique direction, by means of dendriform hubs, to a full pattern which ultimately undergoes deformation due to detachment, from side roads, of new direct roads that gradually attain the rank of the main roads.

The applied procedure of verification comprises two stages: the geographic and the statistical verification. The geographic verification embraces the definition of zones of transport, in which the complexes bear similarity with one or another type or sub-type. The geographic verification, carried out within narrower regional segments, would represent identification of actual complexes subject to further investigation by cartographic and statistical methods as regards their degree of conformity with the theoretical standard. The purpose of the statistical verification is to ascertain whether, and to what degree, the interrelations looked for from a theoretical point of view actually take place. This verification is limited to unalterable features, i.e. to those which fail to be altered by typology alterations and are maintained in historic complexes, in other words, to features characterizing the anisotropic model. In this verification, simple statistic methods have proven to be satisfactory. Their application yielded, for the first feature, a coefficient of concordance of 0.92; for the second feature it is 0.84, and for the third it is 0.77, the standard deviation.

List of Figures

1. Transport network in figures given by J. G. Kohl
2. Pattern of transport network for colonies. Interregional scheme. After W.H. McLean
3. Pattern of transport network. After Cyrus Kehr
4. Hexagonal pattern of transport network. I. Inland scheme. After A. C. Comey
5. Tracks of transport in a system of central foci. After W. Christaller.
6. System of central foci developed on the principle of transport. After W. Christaller
7. Tracks of transport in an ideal economic landscape (one of the sectors). After A. Lösen
8. Modified Lösch system consistent with distribution of population. After W. Isard
9. Transportational effectiveness of various geometric figures
10. Simplified model of complex of transport networks. 1–rivers; 2– railway lines; 3 – roads
11. Cost of investment in 1 km ambidirectional track and 1 ton truck capacity
12. Comparison of net cost of transport of high-value goods by means of various types of transport carriers. After C. Pirath
13. Economic boundaries of circular transport for delivery by 5 t truck, considering both cost and time (Example 1)
14. Economic boundaries of circular transport for delivery by 5 t truck, considering both cost and time (Example 2)
15. Economic boundaries of circular transport for delivery by 5 t truck, considering both cost and time (Example 3)

16. Economic boundaries of circular transport with intermediate stops, for delivery by 5 t truck, considering both cost and time (4 instances of receiving additional loads)
17. Difference in transport rates before and after discontinuation of lateral railway tracks in through-traffic. After C. Pirath.
18. Reverse ratios of volumins and costs of railway and truck transport
19. Optical trend of refracted ray
20. Limit angle
21. Elementary complex. Location of road with regard to railway
22. Elementary complex. Deviation of road due to cost of freight transfer
23. Reorientation of market area (service) due to improvement in transport
24. System of markets along main (improved) transport track
25. Deformation of lateral track in anisotropic complex of transport tracks. Effect of main (improved) track
26. Deformation of pattern of lateral tracks (northeastern sector)
27. Diagram of function versus hexagonal pattern. Comparison of “platination” (flattening effect)
28. Hypothetical anisotropic complex
29. Anisotropic model of complex of transport networks
30. Odotropic triangular complex
31. Odotropic rectangular complex
32. Exotropic triangular complex
33. Exotropic rectangular complex
34. Polycentric triangular complex
35. Polycentric rectangular complex
36. Monocentric triangular complex
37. Monocentric rectangular complex
38. Convergent variety of monocentric rectangular complex