

National core curriculum for mathematics.

Basic level

I. Real Numbers.

The Student:

- performs operations (addition, subtraction, multiplication, division, exponentiation, root extraction, logarithms) in the set of real numbers;
- conducts simple proofs related to the divisibility of integers and remainders of division, not more difficult than:
 - (a) proof of divisibility by 24 of the product of four consecutive natural numbers;
 - (b) proof of the property: if a number leaves a remainder of 3 when divided by 5, then its cube leaves a remainder of 2 when divided by 5;
- applies properties of roots of any degree, including odd-degree roots of negative numbers;
- uses the relationship between root extraction and exponentiation, as well as laws of operations on powers and roots;
- applies the monotonicity of exponentiation, in particular the properties: if $x < y$ and $a > 1$, then $a^x < a^y$; and if $x < y$ and $0 < a < 1$, then $a^x > a^y$;
- uses the concept of a numerical interval, marks intervals on the number line;
- applies both geometric and algebraic interpretations of absolute value, solves equations of the form $|x + 4| = 5$;
- uses the properties of exponentiation and root extraction in practical situations, including to calculate compound interest, investment returns, and credit costs;
- applies the relationship between logarithms and exponentiation, and uses formulas for the logarithm of a product, quotient, and power.

II. Algebraic Expressions.

The Student:

- applies the special product formulas: $(a + b)^2$, $(a - b)^2$, $a^2 - b^2$;
- adds, subtracts, and multiplies polynomials in one or more variables;
- factors out a monomial from an algebraic sum;
- multiplies and divides rational expressions;

III. Equations and Inequalities

The student:

- transforms equations and inequalities in an equivalent way, including, for example, transforming the equation $\frac{5}{x+1} = \frac{x+3}{2x-1}$ equivalently;
- interprets contradictory and identity linear equations and inequalities;
- solves linear inequalities with one unknown;
- solves quadratic equations and inequalities;
- solves polynomial equations of the form $W(x) = 0$, for polynomials expressed in factored form.

IV. Systems of Equations

The student:

- solves systems of linear equations with two variables and provides a geometric interpretation of consistent, inconsistent, and dependent systems;
- applies systems of equations to solve word problems;

V. Functions

The student:

- defines functions as unambiguous assignments using verbal descriptions, tables, graphs, or formulas (including piecewise-defined functions);
- calculates the value at a point of a function given by an algebraic formula;
- reads and interprets function values defined by tables, graphs, formulas, etc., including in situations requiring repeated use of the same or multiple sources of information;

- reads from the graph of a function: the domain, range, zeros, intervals of monotonicity, intervals in which the function takes values greater than (not less than) or less than (not greater than) a given number, and the maximum and minimum values of the function (if they exist) on a given closed interval, as well as the arguments at which those values are attained;
- interprets the coefficients in the formula of a linear function;
- determines the formula of a linear function based on information about its graph or its properties;
- sketches the graph of a quadratic function given by a formula;
- interprets the coefficients in the general, vertex, and factored forms (if it exists) of a quadratic function;
- determines the formula of a quadratic function based on information about the function or its graph;
- determines the maximum and minimum value of a quadratic function on a closed interval;
- uses properties of linear and quadratic functions to interpret geometric, physical, and other problems, including those set in practical contexts;
- based on the graph of a function $y = f(x)$, sketches the graphs of the functions $y = f(x - a)$, $y = f(x) + b$;
- uses the function $f(x) = \frac{a}{x}$, including its graph, to describe and interpret problems involving inverse proportionality, including in practical applications;
- uses exponential and logarithmic functions, including their graphs, to describe and interpret problems involving practical applications.

VI. Sequences

The student:

- calculates terms of a sequence defined by a general formula;
- calculates the initial terms of sequences defined recursively;
- in simple cases, determines whether a sequence is increasing or decreasing;
- checks whether a given sequence is arithmetic or geometric;
- uses the formula for the n th term and the sum of the first n terms of an arithmetic sequence;

- uses the formula for the n th term and the sum of the first n terms of a geometric sequence;
- uses the properties of sequences, including arithmetic and geometric ones, to solve problems, also in practical contexts.

VII. Trigonometry

The student:

- uses the definitions of sine, cosine, and tangent for angles from 0° to 180° , in particular determines the values of trigonometric functions for angles 30° , 45° , and 60° ;
- uses the identities $\sin^2 \alpha + \cos^2 \alpha = 1$ and $\tan \alpha = \frac{\sin \alpha}{\cos \alpha}$;
- applies the law of cosines and the formula for the area of a triangle:
 $P = \frac{1}{2} \cdot a \cdot b \cdot \sin \gamma$;
- calculates angles and side lengths of right triangles given appropriate data (solves right triangles, including using trigonometric functions).

VIII. Plane Geometry (Planimetry)

The student:

- determines the radii and diameters of circles, the lengths of chords and tangents, including using the Pythagorean theorem;
- identifies acute, right, and obtuse triangles given side lengths (e.g., uses the converse of the Pythagorean theorem and the law of cosines); applies the theorem: in a triangle, the longer side lies opposite the greater interior angle;
- identifies regular polygons and uses their basic properties;
- uses the properties of angles and diagonals in rectangles, parallelograms, rhombuses, and trapezoids;
- applies the properties of inscribed and central angles;
- uses formulas for the area of a sector and the length of a circular arc;
- applies Thales' theorem;
- uses the criteria for triangle similarity;
- applies relationships between perimeters and between areas of similar figures;

- identifies key special points in a triangle: incenter, circumcenter, orthocenter, centroid, and uses their properties;
- conducts geometric proofs;
- uses trigonometric functions to determine segment lengths in plane figures and to calculate areas of figures.

IX. Analytic Geometry in the Cartesian Plane

The student:

- identifies the relative positions of lines in the plane based on their equations, including finding the intersection point of two lines, if it exists;
- uses equations of lines in the plane, in both slope-intercept and general form, including determining the equation of a line with given properties (such as passing through two given points, known slope, parallel to another line, or tangent to a circle);
- calculates the distance between two points in the coordinate system;
- uses the equation of a circle $(x - a)^2 + (y - b)^2 = r^2$;
- determines the images of circles and polygons under axial symmetries with respect to the coordinate axes and under central symmetry (with center at the origin of the coordinate system).

X. Solid Geometry (Stereometry)

The student:

- identifies the relative positions of lines in space, in particular skew perpendicular lines;
- uses the concept of the angle between a line and a plane, as well as the dihedral angle between half-planes;
- identifies angles between segments (e.g., edges, edges and diagonals) and angles between faces in prisms and pyramids, and calculates the measures of these angles;
- identifies angles between segments, and between segments and planes in cylinders and cones (e.g., cone opening angle, angle between a slant height and the base), and calculates the measures of these angles;
- calculates the volumes and surface areas of prisms, pyramids, cylinders, cones, and spheres, including with the use of trigonometry and known theorems;
- uses the relationship between the volumes of similar solids.

XI. Combinatorics

The student:

- counts objects in simple combinatorial situations;
- counts objects using the multiplication and addition principles (also in combination) for any number of actions in situations no more difficult than:
 - (a) calculating how many four-digit positive odd integers there are such that their decimal representation contains exactly one digit 1 and exactly one digit 2,
 - (b) calculating how many four-digit positive even integers there are such that their decimal representation contains exactly one digit 0 and exactly one digit 1;

XII. Probability and Statistics

The student:

- calculates probabilities in the classical model;
- calculates the arithmetic mean and the weighted mean, finds the median and the mode;

XIII. Optimization

The student:

- solves optimization problems in situations that can be described using a quadratic function.